201 §2.3 #

1 Find eq'n of tan line to the curve @ the given point.

$y = \frac{2x}{x+1}$ @ $(1,1)$: $y' = \frac{2(x+1) - (2x)(1)}{(x+1)^2} \Longrightarrow$

$y'(1) = \frac{2(2) - 2}{2^2} = \frac{4-2}{4} = \frac{1}{2} = m$

$\boxed{y = \frac{1}{2}(x-1) + 1}$ $\quad y = m(x - x_1) + y_1$

2 $y = x^4 + 2x^2 - x$ @ $(1,2)$ $\longrightarrow$

$y' = 4x^3 + 4x - 1 \Longrightarrow$

$y'(1) = 4 + 4 - 1 = 7 \Longrightarrow$

$\boxed{y = 7(x-1) + 2}$

Find tan line and <u>NORMAL LINE</u>

3 $y = x + \sqrt{x} = x + x^{\frac{1}{2}}$ @ $(1, 2)$

$y' = 1 + \frac{1}{2}x^{-\frac{1}{2}} \Longrightarrow y'(1) = 1 + \frac{1}{2} = \frac{3}{2} = m_{tan}$

$\longrightarrow m_{\perp} = -\frac{2}{3}$

TAN LINE : $y = \frac{3}{2}(x-1) + 2$

NORMAL LINE : $y = -\frac{2}{3}(x-1) + 2$

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4) $y = \dfrac{3x+1}{x^2+1}$ @ $(1,2)$:

$$y' = \frac{3(x^2+1) - (3x+1)(2x)}{(x^2+1)^2} \Longrightarrow$$

$$y'(1) = \frac{3(2) - (4)(2)}{2^2} = \frac{6-8}{4} = -\frac{2}{4} = -\frac{1}{2} = m_{tan}$$

$\Longrightarrow m_\perp = 2 \Longrightarrow$

TAN LINE : $y = -\frac{1}{2}(x-1) + 2$

NORMAL LINE : $y = 2(x-1) + 2$

Find 1st & 2nd Derivatives

5) $f(x) = x^4 - 3x^3 + 16x \Longrightarrow$

$f'(x) = 4x^3 - 9x^2 + 16 \Longrightarrow$

$f''(x) = 12x^2 - 18x$

6) Eq'n of motion is $s = t^3 - 3t$, where s is in m and t is in s.

(a) Find velocity & acceleration func's

$v(t) = 3t^2 - 3$, $a(t) = 6t$

(b) $\boxed{a(1) = 6}$

(c) Graph

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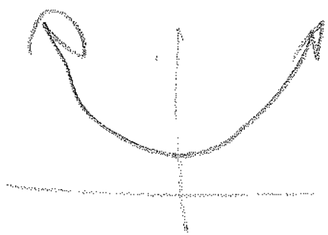
7 Show that $6x^3+5x-3$ has no tan line with $m=4$.

$f'(x)=12x^2+5 \overset{SET}{=} 4 \Longrightarrow$

$12x^2+1=0 \Longrightarrow$ Never!



$12x^2+1>0 \ \forall x$

$\Longleftrightarrow$

$12x^2+5=m=4$ has no soln.

8 Find eqn of normal line to parabola $y=x^2-5x+4$ that's $\perp$ to $x-3y=5$

No. Want Parallel to that line.

$x-3y=5 \longrightarrow -3y=-x+5$

$\Longrightarrow y=\frac{1}{3}x+\frac{5}{3} \Longrightarrow m=\frac{1}{3} \Longrightarrow m_\perp=-3$

$y'=2x-5 \overset{SET}{=} -3$

Hmmm. Tricky. This might be right, in spite of me. Wanting a normal to the curve that's parallel to the line. I think that's the same as finding a spot where the tangent to the curve is *perpendicular* to the line.
Yup. I think I got it right, in spite of myself.

$2x=2$

$\boxed{x=1}$

$\Longrightarrow y=1-5+4=0$

$\Longrightarrow \boxed{y=-3(x-1)+0}$

or $y=-3x+3$

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9) § $f(5)=1,\ f'(5)=6,\ g(5)=-3,\ g'(5)=2$

(a) $(fg)'(5) = f'(5)g(5) + f(5)g'(5)$

$= (6)(-3) + (1)(2) = -18+2 = \boxed{-16}$

(b) $\left(\frac{f}{g}\right)'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{(g(5))^2}$

$= \frac{(6)(-3) - (1)(2)}{(-3)^2} = \frac{-18-2}{9} = \boxed{-\frac{20}{9}}$

(c) $\left(\frac{g}{f}\right)'(5) = \frac{g'(5)f(5) - g(5)f'(5)}{(f(5))^2}$

$= \frac{(2)(1) - (-3)(6)}{1^2} = 2+18 = \boxed{20}$

10) If $f(x) = \sqrt{x}\, g(x)$, $g(4) = 8$ & $g'(4) = 7$,

find $f'(4)$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} g(x) + x^{\frac{1}{2}} g'(x) \Rightarrow$

$f'(4) = \frac{1}{2}(4)^{-\frac{1}{2}} g(4) + 4^{\frac{1}{2}} g'(4)$

$= \frac{1}{2\cdot 2}\cdot 8 + 2\cdot 7 = 2+14 = \boxed{16}$

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11 Find a cubic func. $y = ax^3 + bx^2 + cx + d$ $\ni$ it has horizontal tangents @ $(-2,6)$ & $(2,0)$

$y' = 3ax^2 + 2bx + c \overset{SET}{=} 0$ Hmmmmm

$f(2) = 0 \Longrightarrow f(x) = a(x-2)(x^2 + bx + c)$

$f(-2) = 6 \Longrightarrow f(-2) = -4a(4 - 2b + c) = 6$

$-16a + 8ab - 4ac = 6$

Hmmmm

Let's go horizontal tangents route.

$y' = 3a(x-2)(x+2) \quad (y' = 0 \text{ @ } x = \pm 2)$

$= 3a(x^2 - 4)$

$= 3ax^2 - 12a = 3ax^2 + 2bx + c$

$\Longrightarrow b = 0 \quad \& \quad c = -12a$

$\Longrightarrow y = ax^3 - 12ax + d$

$y(2) = 0 \Longrightarrow 8a - 24a + d = 0$

$-16a + d = 0$

$-16a = -d$

$a = \frac{1}{16} d$

$y(-2) = 6 \Longrightarrow \frac{1}{16} d(-2)^3 - 12\left(\frac{1}{16} d\right) + d = 6$

$-\frac{1}{2} d - \frac{3}{4} d + d = 6$

$-2d - 3d + 4d = 24$

$-d = 24$

$d = -24$

$a = \frac{1}{16} \cdot 24 = \frac{3}{2}$

$\boxed{y = \frac{3}{2} x^3 - 6x - 24}$