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1 Write an eq'n for the secant line's slope through $P(3, f(3))$ and $Q(x, f(x))$:

1a $m = \frac{y_2 - y_1}{x_2 - x_1} = \boxed{\frac{f(x) - f(3)}{x-3} = m_{sec}}$

1b Write expression for slope of tan. line @ $x = 3$:

$$m_{tan} = f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x-3}$$

2 Find slope of tan. line to $y = 4x - x^2$

2a @ $(x_1, y_1) = (1, 3)$:

$$m_{sec} = \frac{f(x) - f(1)}{x-1} = \frac{4x - x^2 - (4(1) - 1^2)}{x-1}$$

$$= \frac{4x - x^2 - 3}{x-1} = \frac{-x^2 + 4x - 3}{x-1} = -\frac{(x^2 - 4x + 3)}{x-1}$$

$$= -\frac{(x-1)(x-3)}{x-1} = -(x-3) \xrightarrow{x \to 1} -(1-3) = \boxed{2 = m_{tan}}$$

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Alternate:

2a $f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$:

$$\frac{f(1+h) - f(1)}{h} = \frac{4(1+h) - (1+h)^2 - (3)}{h}$$

$$= \frac{4 + 4h - (1 + 2h + h^2) - 3}{h} = \frac{4 + 4h - 1 - 2h - h^2 - 3}{h}$$

$$= \frac{2h - h^2}{h} = 2 - h \xrightarrow{h \to 0} \boxed{2 = m_{tan}}$$

2b Find eq'n of tan. line

$y = m(x - x_1) + y_1$

$\boxed{y = 2(x-1) + 3}$

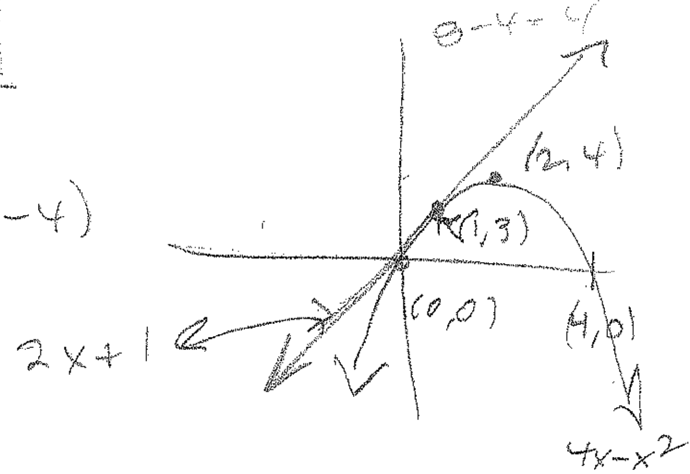
$4 - 2x = 0$

$x = 2$

$8 - 4 = 4$

2c $4x - x^2 = -x(x-4)$

Sketch



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3a) Find slope of tangent line to parabola $y = 4x - x^2$ @ $(x_1, y_1) = (1, 3)$.

This ain't the most efficient way, but

$\boxed{M1}$ $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$ OR $\boxed{M2}$ $\lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x-1}$

are equivalent for finding m_{tan} at $x = 1$.

Book's stressing $\boxed{M2}$, so I follow suit:

$$\frac{f(x) - f(1)}{x-1} = \frac{4x - x^2 - 3}{x-1} = \frac{-(x^2 - 4x + 3)}{x-1}$$

$$= \frac{-(x-3)(x-1)}{x-1} \underset{(x \neq 1)}{=} -(x-3) \xrightarrow{x \to 1} -(1-3) = +2$$

$$\boxed{m_{tan} = 2}$$

3b An eq'n of tangent line @ (1, 3) is

$y = m_{tan}(x - x_1) + y_1$

$$\boxed{y = 2(x-1) + 3} = 2x - 2 + 3 = \underline{2x + 1 = y}$$

Better than book. Book

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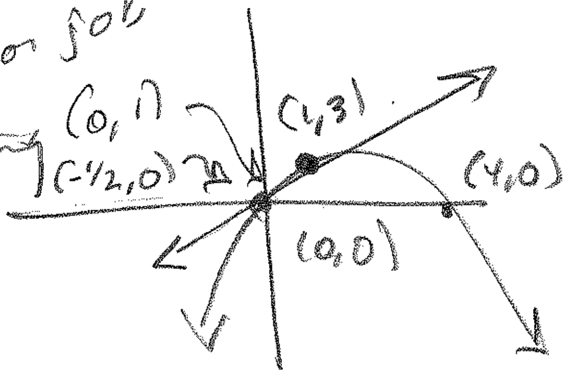
-4B

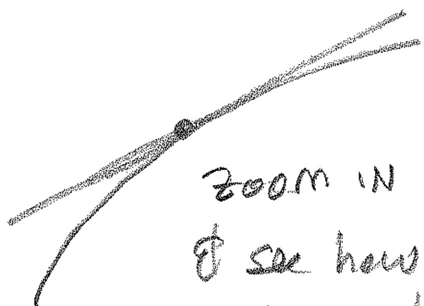
3c

c. &

Poor job showing



tan. line; poorly. This grapher exercise/exploration. The idea is, the more you zoom in, the more the function & tan. line look the same.



ZOOM IN & see how close they stay?

SMOOTH CURVES ARE "LOCALLY LINEAR"

4a

We find the slope of the tan. line to $y = x - x^3$ @ $(1, 0)$.

$$m_{sec} = \frac{f(x) - f(1)}{x - 1} = \frac{x - x^3 - 0}{x - 1} = \frac{-(x^3 - x)}{x - 1}$$

$$= \frac{-x(x^2 - 1)}{x - 1} = \frac{-x(x-1)(x+1)}{x - 1} = \frac{-x(x+1)}{1}$$

$(x \neq 1)$

$$x \longrightarrow 1 \longrightarrow \frac{-1(1+1)}{1} = \boxed{-2 = m_{tan}}$$

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4b

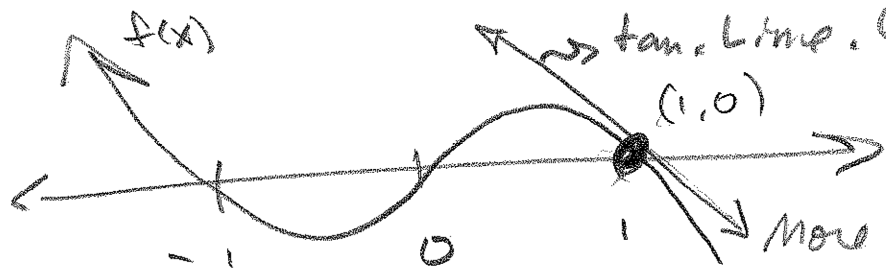
Eq'n of tan. line :

$y = m(x - x_1) + y_1$

$= -2(x-1) + 0$

4c

Grapher Exploration. Idea from a hand sketch : $-x(x^2-1) = -x(x-1)(x+1)$



More you zoom in, the more alike the two are.

#s 5-8 Find an eq'n of tangent line to the curve @ the given point

5

$y = \sqrt{x}$ @ (1, 1)

(m) $\frac{f(x) - f(1)}{x-1} = \frac{\sqrt{x} - 1}{x - 1} = \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{1}{\sqrt{x}+1}$, $x \neq 1$

$\xrightarrow{x \to 1} \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2} = m_{tan}}$

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5 cont'd

$\boxed{M2}$ $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \left(\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) = \dfrac{\sqrt{x+h}^2-\sqrt{x}^2}{h(\sqrt{x+h}+\sqrt{x})}$

$= \dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \dfrac{h}{h(\sqrt{x+h}+\sqrt{x})} = \dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

$(h \neq 0)$

$\xrightarrow{h\to 0} \dfrac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\dfrac{1}{2\sqrt{x}} = f'(x)}$

So, plug in $x=1$: $f'(1) = \boxed{\frac{1}{2} = m_{tan}}$

$\boxed{M1}$ More details:

$\dfrac{\sqrt{x}-1}{x-1} = \left(\dfrac{\sqrt{x}-1}{x-1}\right)\left(\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right) = \dfrac{\sqrt{x}^2-1^2}{(x-1)(\sqrt{x}+1)}$

$= \dfrac{x-1}{(x-1)(\sqrt{x}+1)} = \dfrac{1}{\sqrt{x}+1} \xrightarrow{x\to 1} \dfrac{1}{\sqrt{1}+1} = \dfrac{1}{2}$

$x \neq 1$

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5 cnt'd. The 1^{st} way I worked this, I simply viewed $x-1$ as a difference of 2 squares!

$$x-1=\sqrt{x}^{2}-1^{2}=(\sqrt{x}-1)(\sqrt{x}+1)$$

which is kinda cool, but I figured it'd freak people out!

YET ANOTHER WAY:

Find $f'(x)$, plug in specific x, using

$$f'(x)=\lim_{c\to x}\frac{f(x)-f(c)}{x-c}=\lim_{c\to x}\frac{\sqrt{x}-\sqrt{c}}{x-c}= \text{same tricksies}$$

$$=\lim_{c\to x}\frac{x-c}{(x-c)(\sqrt{x}+\sqrt{c})}=\lim_{c\to x}\frac{1}{\sqrt{x}+\sqrt{c}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}},$$

ANOTHER way:

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\dots=\frac{1}{2\sqrt{c}}$$

they're all the same idea, when you see

you use whichever one works better in a given situation.

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5 FINISH!

$m_{tan} = \frac{1}{2}$, $(x_1, y_1) = (1,1) \Longrightarrow$

Tan. line is given by $\boxed{y = \frac{1}{2}(x-1)+1}$

This is also sometimes called the "Linearization" $L(x)$ for $f(x)$ @ $x=1$

$$L_1(x) = \frac{1}{2}(x-1)+1$$

6 $y = \frac{2x+1}{x+1}$ @ $(1,1)$

$\boxed{M1}$ Book way: $(a, f(a)) = (1,1)$

$$\cancel{\frac{f(a+h)-f(a)}{h} = \frac{2(1+h)+1-(2(1)+1)}{h}}$$

$$\cancel{= \frac{2+2h+1-2-1}{h} =}$$

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{2(1+h)+1}{(1+h)+1} - \frac{2(1)+1}{1+1}}{h}$$

$$= \frac{\frac{2+2h+1}{1+h+1} - \frac{3}{2}}{h} = \frac{\frac{2h+3}{h+2} - \frac{3}{2}}{h}$$

$$= \frac{\frac{2h+3}{h+2}\cdot\frac{2}{2} - \frac{3}{2}\cdot\frac{h+2}{h+2}}{h} = \frac{\frac{4h+6-(3h+6)}{2(h+2)}}{h}$$

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6 cont'd

$$= \frac{\frac{4h+6-3h-6}{2(h+2)}}{h} = \frac{\frac{h}{2(h+2)}}{h} = \frac{\frac{h}{2(h+2)}}{\frac{h}{1}}$$

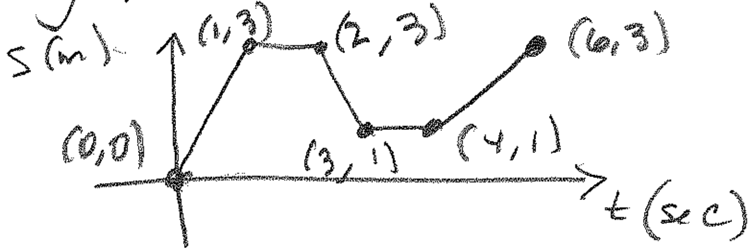
$$= \left(\frac{h}{2(h+2)}\right)\left(\frac{1}{h}\right) = \frac{1}{2(h+2)} \xrightarrow{h \to 0} \boxed{\frac{1}{4}} = m_{tan}$$

$$y = m(x - x_1) + y_1$$

$$\boxed{y = \frac{1}{4}(x-1) + 1}$$

I hate these exercises. Bad enough using the limit definition, but to plug in $x=1$ @ 1st step, instead of just using the variable x is painful 4 me.

7 A particle starts by moving right along a horizontal line. Its position (Distance from its starting point) is shown



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11 cntd

7a The particle is moving left from $t=4$ to $t=6$, or $\forall t \in (4,6)$.. (i.e. $4<t<6$)

Moving right $\forall t \in (0,1) \cup (4,6)$

Standing still $\forall t \in (1,2) \cup (3,4)$

I'm not too worried, here, about $(1,2)$ -vs- $[1,2]$.

7b A graph of velocity function:

on $(0,1)$: $m = \frac{3-0}{1-0} = 3$

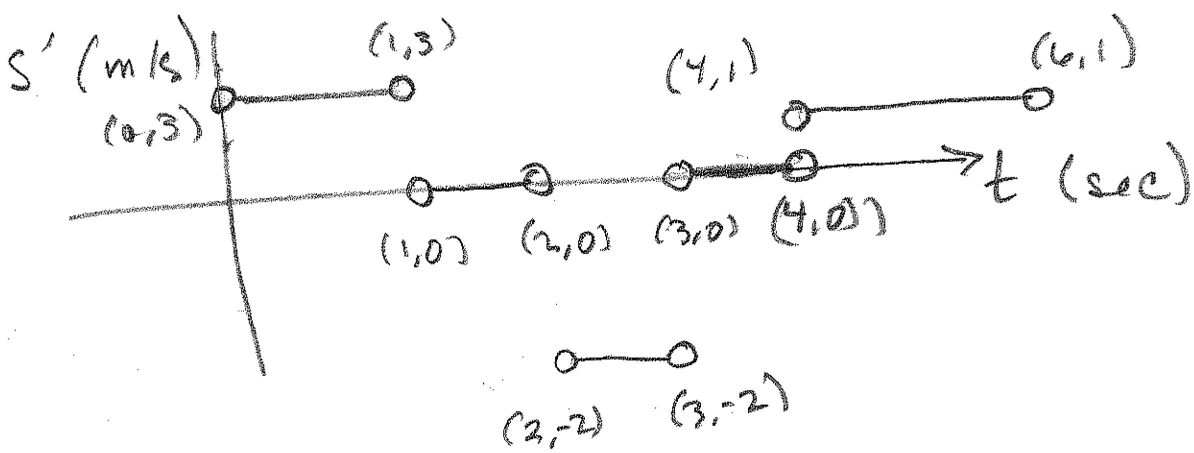
on $(1,2)$: $m = 0$

on $(2,3)$: $m = \frac{1-3}{3-2} = \frac{-2}{1} = -2$

on $(3,4)$: $m = 0$

on $(4,6)$: $m = \frac{3-1}{6-4} = \frac{2}{2} = 1$

Sketch of the velocity func:



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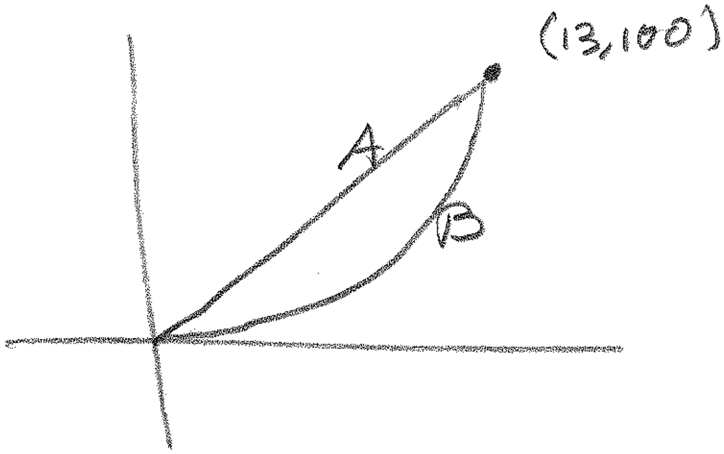
8 Position Functions of two runners who finished in a tie:



8a Looks like B started slow but has strong kick. A ran @ same speed, start to finish.

8b The distance between them is greatest at the same time B starts running at the same speed, so answer to (c) & (b) is the same: About 8 or 9 seconds?

B is farthest behind RIGHT when he stops falling further behind, which is RIGHT when their speeds are the same!

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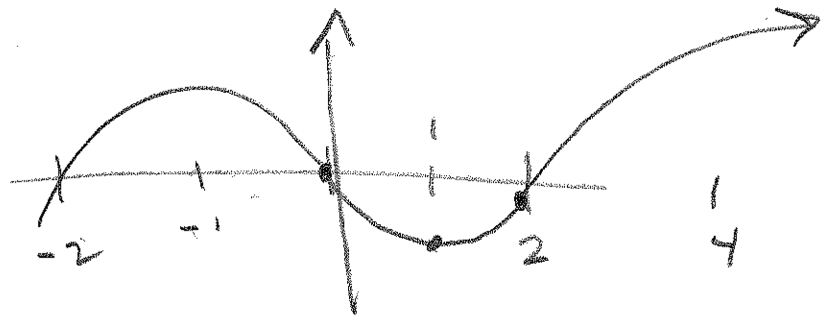
9 For the function g in the graph, arrange the following in increasing order.

$0, g'(-2), g'(0), g'(2), g'(4)$



$g'(0)$, 0 , $g'(4)$, $g'(2)$

most negative slope

positive, but less than $g'(2)$

most positive slope

Each limit defines the derivative of some f @ some a. Tell me f & a.

10 $\displaystyle\lim_{h\to 0} \frac{(1+h)^{10}-1}{h}$ $\boxed{f(x)=x^{10} \text{ @ } a=1}$ $f'(1)$

11 $\displaystyle\lim_{x\to 5} \frac{2^x-32}{x-5}$ $\boxed{f(x)=2^x \text{ @ } a=5}$ $f'(5)$