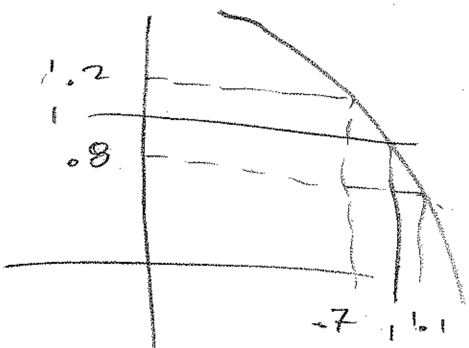
201 §1.7

1



Use graph to find δ corresponding to $\varepsilon = 0.2$

Want $\delta \ni 0 < |x-1| < \delta$

$\Longrightarrow |f(x) - 1| < 0.2$

Let $\boxed{\delta = 0.1}$ ∴ $0.9 < x < 1.1$ will keep $|f(x) - 1| < \varepsilon = .2$

2 Want disk $\ni A = \pi x^2 = 1000 \Longrightarrow$

(a) radius is found by solving

$\pi x^2 = 1000$

$$x^2 = \frac{1000}{\pi}$$

$$x = \pm \frac{10\sqrt{10}}{\sqrt{\pi}} \text{ or } \frac{10\sqrt{10\pi}}{\pi}$$

Ditch the negative.

b) If error is to be within $\pm 5 cm^2$ of $1000 cm^2$

How close to $\frac{10\sqrt{10\pi}}{\sqrt{\pi}}$ must the radius be?

Want $|\pi x^2 - 1000| < 5 \longrightarrow$

$-5 < \pi x^2 - 1000 < 5 \Longrightarrow$

$995 < \pi x^2 < 1005$

for § 1.7

2 cont'd

$$\Longrightarrow \frac{995}{\pi} < x^2 < \frac{1005}{\pi}$$

$$\Longrightarrow \sqrt{\frac{995}{\pi}} < x < \sqrt{\frac{1005}{\pi}}$$

$$\Longrightarrow \sqrt{\frac{995}{\pi}} - \frac{10\sqrt{10\pi}}{\pi} < x - \frac{10\sqrt{10\pi}}{\pi} < \frac{\sqrt{1005}}{\pi} - \frac{10\sqrt{10\pi}}{\pi}$$

Pick the smalles in absolute value of

$$\left| \frac{\sqrt{1005\pi}}{\pi} - \frac{10\sqrt{10\pi}}{\pi} \right| \approx .044547488$$

and

$$\left| \frac{\sqrt{995\pi}}{\pi} - \frac{10\sqrt{10\pi}}{\pi} \right| \approx .0446589966$$

$.044547488 = \delta$ is small enough tolerance to keep area within $5\,cm^2$ of desired $1000\,cm^2$.

4 $\lim_{x \to -3} (1-4x) = 13$

#3 got stuck at the end. Retro-fit. And don't *you* pull that on *your* homework.

$\boxed{\text{Pf}}$ Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{4}$. Then,

if $0 < |x+3| < \delta$, we have $|(1-4x) - 13|$

$= |-12-4x| = |4x+12| = 4|x+3| < 4\delta = 4\,\frac{\varepsilon}{4} = \varepsilon$ ■

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5 $\lim_{x \to -2} (3x+5) = -1$

Pf

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$. Then, if $0 < |x+2| < \delta$, we have $|3x+5-(-1)|$

$= |3x+6| = 3|x+2| < 3\delta = 3\frac{\varepsilon}{3} = \varepsilon$ ∎

6 $\lim_{x \to 2} (x^2-4x+5) = 1$

SCRATCH:

$|x^2-4x+5-1| < \varepsilon$

$|x^2-4x+4| < \varepsilon$

$|x-2|^2 < \varepsilon$

$|x-2| < \sqrt{\varepsilon}$

$\delta = \sqrt{\varepsilon}$ works

Pf Let $\varepsilon > 0$ be given. Define $\delta = \sqrt{\varepsilon}$. Then, if $0 < |x-2| < \delta$, we have

$|x^2-4x+5-1| = |x^2-4x+4|$

$= |x-2|^2 < \delta^2 = (\sqrt{\varepsilon})^2 = \varepsilon$ ∎

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7 $\lim_{x \to 3} x^3 = 8$ is the claim.

Scratch

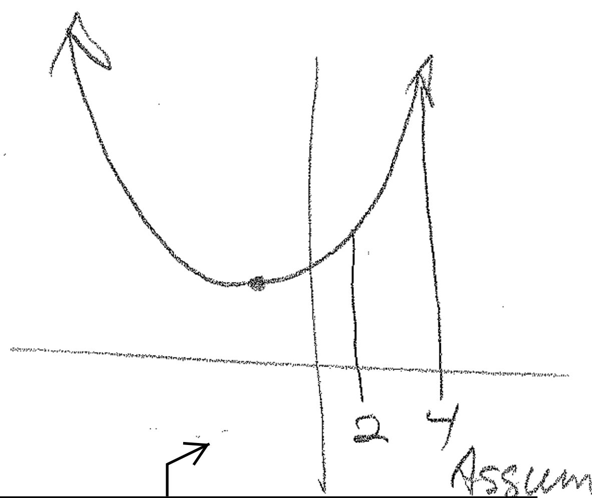
$x^3 - 8 = x^3 - 2^2 = (x-2)(x^2+2x+4)$

is gonna be $< \delta(x^2+2x+4)$, so we need a handle on x^2+2x+4.

Quick pic of $x^2+2x+4 = x^2+2x+1-1+4$

$= (x+1)^2+3$

$(h,k) = (-1,3)$



We're interested in the vicinity of $x=3$. Assuming $\delta \le 1$, we have

$2 < x < 4$ whenever

$|x-3| < 1$

This is all messed up, because I confused a '3' with a '2' and used one then the other, interchangeably. I'm attaching lecture notes with limit as x approaches 3 version of this, to the end of this document, and you should do limit as x approaches 2, with this as a guide.

Since x^2+2x+4 is increasing we see the biggest it can get is $4^2+2(4)+4$, when $x=4$, i.e.

$2 < x < 4 \Longrightarrow |x^2+2x+4| < \boxed{28}$. Now

Write proof!

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Pf (32)

<u>Proof</u> Let $\varepsilon > 0$ be given, Define $\delta = \min\{1, \frac{\varepsilon}{28}\}$. Then any time we have $0 < |x-3| < \delta$, we will also have $|x^3 - 8| = |x-2||x^2+2x+4|$

$$< |x^2+2x+4|\,\delta < 28\,\delta \leq 28 \cdot \frac{\varepsilon}{28} = \varepsilon \; \blacksquare$$

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8 $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ is the claim.

Scratch want $|\sqrt{x} - \sqrt{a}| < \varepsilon$

$$\Rightarrow \frac{|\sqrt{x} - \sqrt{a}|}{1}\left(\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right) = \frac{||x - a|}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{x}+\sqrt{a}}$$

want that $< \varepsilon$.

want a string of "=" and "<"
ending with "$<\varepsilon$". Hmmm mmm.

$$\frac{\delta}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{a}} \overset{\text{WANT}}{<} \varepsilon \Longrightarrow$$

I'm saying this somewhat imprecisely. We'll get our string of "=" with at least one "<" between start and finish if we let delta EQUAL that thing on the right, as in proof.

$$\delta < \varepsilon\sqrt{a}$$

That's it!

Proof Let $\varepsilon > 0$ be given. Define $\delta = \varepsilon\sqrt{a}$

Then if $0 < |x-a| < \varepsilon$ we have

Should be delta.

$$|\sqrt{x} - \sqrt{a}| = \frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \frac{|x-a|}{\sqrt{a}} < \frac{\delta}{\sqrt{a}} = \frac{\varepsilon\sqrt{a}}{\sqrt{a}}$$

$$= \varepsilon \blacksquare$$

Since square root of x is positive, we're making the denominator a little smaller, which makes the fraction a little bigger, so the one on the left is smaller than the one on the right.

201 § 4.7 #3

(3) Temperature T is a function of power, w, given by $T = T(w) = 0.1w^2 + 2.155w + 20$.

Units: T: degrees Celsius

w: Watts.

(a) How much power to maintain temp @ 200^0 C?

$T = 200 \implies$

$.1w^2 + 2.155w + 20 = 200 \implies$

$.1w^2 + 2.155w - 180 = 0$

$100w^2 + 2155w + 180{,}000 = 0$

$b^2 - 4ac = 2155^2 - 4(100)(-180\,000)$

$= 76644025 \longrightarrow$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2155 \pm \sqrt{76644025}}{2(100)}$$

$\approx \boxed{32.9982867 \text{ Watts}}$ Discard negative answer.

So let's call it 32 for the sequel

32.00, in fact

The other sol'n is -54.5482867

Call it -54.55, for practical..

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(3) cont'd

(b) Temp is allowed to vary as much as $\pm 1°C$. What's the range in wattage (power) allowed, then?

Want $|T-200| < 1$

By Previous Work,

$$|T-200| \approx |(W-33.00)(W+54.55)|$$

$$= \underbrace{|W-33.00|}_{\text{Want to know how big this can be}}\,|W+54.55| \underline{< 1}$$

our goal

So it's a question of how big $|W+54.55|$ can be! Let's assume $|W-33.00| < 1$ (We'll be at LEAST that close to 33 Watts)

Then $32 < W < 34$ and so

$$32+54.55 < W+54.55 < 34+54.55$$

and so $W+54.55 < 88.55$ USE this

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(3) cont'd

$$|w+54.55| < 88.55 \longrightarrow$$

$$|w-33.001| + 54.55| < |w-33|(89)$$

So $(89)|w-33| < 1 \longrightarrow$

$$|w-33| < \frac{1}{89}.$$

This says, to keep w between $33-\frac{1}{89}$ and $33+\frac{1}{89}$

Notice I rounded the 88.55 <u>up</u>. This was a conservative move. Making

$|w-33||w+88.55|$ a little bigger:

$|w-33||w+89|$ made us keep $|w-33|$ a little smaller, to satisfy

$$|w-33||w+89| < 1$$