1

(a) $\lim_{x\to 2}[f(x)+5g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2}[5g(x)]$ [Limit Law 1]

$= \lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x)$ [Limit Law 3]

$= 4 + 5(-2) = -6$

(b) $\lim_{x\to 2}[g(x)]^3 = \left[\lim_{x\to 2} g(x)\right]^3$ [Limit Law 6]

$= (-2)^3 = -8$

(c) $\lim_{x\to 2}\sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)}$ [Limit Law 11]

$= \sqrt{4} = 2$

(d) $\lim_{x\to 2}\frac{3f(x)}{g(x)} = \frac{\lim_{x\to 2}[3f(x)]}{\lim_{x\to 2} g(x)}$ [Limit Law 5]

$= \frac{3\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$ [Limit Law 3]

$= \frac{3(4)}{-2} = -6$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim_{x\to 2}\frac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $\lim_{x\to 2}\frac{g(x)\,h(x)}{f(x)} = \frac{\lim_{x\to 2}[g(x)\,h(x)]}{\lim_{x\to 2} f(x)}$ [Limit Law 5]

$= \frac{\lim_{x\to 2} g(x)\cdot\lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)}$ [Limit Law 4]

$= \frac{-2\cdot 0}{4} = 0$

2

(a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x)\cdot\lim_{x\to 0} g(x) = 0\cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3\cdot 2 = 16$

(f) $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

3
$$\lim_{t\to-2}\frac{t^4-2}{2t^2-3t+2}=\frac{\lim\limits_{t\to-2}(t^4-2)}{\lim\limits_{t\to-2}(2t^2-3t+2)} \qquad \text{[Limit Law 5]}$$

$$=\frac{\lim\limits_{t\to-2}t^4-\lim\limits_{t\to-2}2}{2\lim\limits_{t\to-2}t^2-3\lim\limits_{t\to-2}t+\lim\limits_{t\to-2}2} \qquad \text{[1, 2, and 3]}$$

$$=\frac{16-2}{2(4)-3(-2)+2} \qquad \text{[9, 7, and 8]}$$

$$=\frac{14}{16}=\frac{7}{8}$$

4
$$\lim_{x\to2}\sqrt{\frac{2x^2+1}{3x-2}}=\sqrt{\lim_{x\to2}\frac{2x^2+1}{3x-2}} \qquad \text{[Limit Law 11]}$$

$$=\sqrt{\frac{\lim\limits_{x\to2}(2x^2+1)}{\lim\limits_{x\to2}(3x-2)}} \qquad \text{[5]}$$

$$=\sqrt{\frac{2\lim\limits_{x\to2}x^2+\lim\limits_{x\to2}1}{3\lim\limits_{x\to2}x-\lim\limits_{x\to2}2}} \qquad \text{[1, 2, and 3]}$$

$$=\sqrt{\frac{2(2)^2+1}{3(2)-2}}=\sqrt{\frac{9}{4}}=\frac{3}{2} \qquad \text{[9, 8, and 7]}$$

5

(a) The left-hand side of the equation is not defined for $x=2$, but the right-hand side is.

(b) Since the equation holds for all $x\neq2$, it follows that both sides of the equation approach the same limit as $x\to2$, just as in Example 3. Remember that in finding $\lim\limits_{x\to a}f(x)$, we never consider $x=a$.

6
$$\lim_{x\to5}\frac{x^2-6x+5}{x-5}=\lim_{x\to5}\frac{(x-5)(x-1)}{x-5}=\lim_{x\to5}(x-1)=5-1=4$$

7
$\lim\limits_{x\to5}\dfrac{x^2-5x+6}{x-5}$ does not exist since $x-5\to0$, but $x^2-5x+6\to6$ as $x\to5$.

8
$$\lim_{x\to-4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x}=\lim_{x\to-4}\frac{\frac{x+4}{4x}}{4+x}=\lim_{x\to-4}\frac{x+4}{4x(4+x)}=\lim_{x\to-4}\frac{1}{4x}=\frac{1}{4(-4)}=-\frac{1}{16}$$

9 $\lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} = \lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} \cdot \frac{\sqrt{1+t} + \sqrt{1-t}}{\sqrt{1+t} + \sqrt{1-t}} = \lim_{t \to 0} \frac{\left(\sqrt{1+t}\right)^2 - \left(\sqrt{1-t}\right)^2}{t\left(\sqrt{1+t} + \sqrt{1-t}\right)}$

$$= \lim_{t \to 0} \frac{(1+t) - (1-t)}{t\left(\sqrt{1+t} + \sqrt{1-t}\right)} = \lim_{t \to 0} \frac{2t}{t\left(\sqrt{1+t} + \sqrt{1-t}\right)} = \lim_{t \to 0} \frac{2}{\sqrt{1+t} + \sqrt{1-t}}$$

$$= \frac{2}{\sqrt{1} + \sqrt{1}} = \frac{2}{2} = 1$$

10 $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$$

11 $-1 \le \cos(2/x) \le 1 \quad \Rightarrow \quad -x^4 \le x^4 \cos(2/x) \le x^4$. Since $\lim_{x \to 0} (-x^4) = 0$ and $\lim_{x \to 0} x^4 = 0$, we have $\lim_{x \to 0} \left[x^4 \cos(2/x)\right] = 0$ by the Squeeze Theorem.

12 $\left|2x^3 - x^2\right| = \left|x^2(2x-1)\right| = \left|x^2\right| \cdot |2x-1| = x^2\,|2x-1|$

$$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \ge 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \ge 0.5 \\ -(2x-1) & \text{if } x < 0.5 \end{cases}$$

So $\left|2x^3 - x^2\right| = x^2[-(2x-1)]$ for $x < 0.5$.

Thus, $\lim_{x \to 0.5^-} \frac{2x-1}{\left|2x^3 - x^2\right|} = \lim_{x \to 0.5^-} \frac{2x-1}{x^2[-(2x-1)]} = \lim_{x \to 0.5^-} \frac{-1}{x^2} = \frac{-1}{(0.5)^2} = \frac{-1}{0.25} = -4.$

13 Since $|x| = -x$ for $x < 0$, we have $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{-x}\right) = \lim_{x \to 0^-} \frac{2}{x}$, which does not exist since the denominator approaches 0 and the numerator does not.

14 Since $|x| = x$ for $x > 0$, we have $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x}\right) = \lim_{x \to 0^+} 0 = 0.$

15 (a) (i) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \frac{x^2+x-6}{|x-2|} = \lim_{x\to 2^+} \frac{(x+3)(x-2)}{|x-2|}$

$= \lim_{x\to 2^+} \frac{(x+3)(x-2)}{x-2}$ [since $x-2>0$ if $x\to 2^+$]

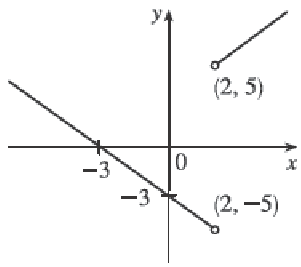
$= \lim_{x\to 2^+} (x+3) = 5$

(ii) The solution is similar to the solution in part (i), but now $|x-2| = 2-x$ since $x-2<0$ if $x\to 2^-$.

Thus, $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} -(x+3) = -5$.

(b) Since the right-hand and left-hand limits of g at $x=2$ are not equal, $\lim_{x\to 2} g(x)$ does not exist.

(c)



16 $\lim_{v\to c^-} \left(L_0\sqrt{1-\frac{v^2}{c^2}}\right) = L_0\sqrt{1-1} = 0$. As the velocity approaches the speed of light, the length approaches 0.

A left-hand limit is necessary since L is not defined for $v>c$.

17 (a) $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x^2\right] = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x^2 = 5\cdot 0 = 0$

(b) $\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\left[\frac{f(x)}{x^2}\cdot x\right] = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x = 5\cdot 0 = 0$

18 Since the denominator approaches 0 as $x\to -2$, the limit will exist only if the numerator also approaches 0 as $x\to -2$. In order for this to happen, we need $\lim_{x\to -2}\left(3x^2+ax+a+3\right) = 0 \Leftrightarrow$

$3(-2)^2 + a(-2) + a + 3 = 0 \Leftrightarrow 12 - 2a + a + 3 = 0 \Leftrightarrow a = 15$. With $a=15$, the limit becomes

$\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2}\frac{3(x+2)(x+3)}{(x-1)(x+2)} = \lim_{x\to -2}\frac{3(x+3)}{x-1} = \frac{3(-2+3)}{-2-1} = \frac{3}{-3} = -1.$