201 §1.5

1 $\lim_{x \to 2} f(x) = 5$ means... Answers vary.

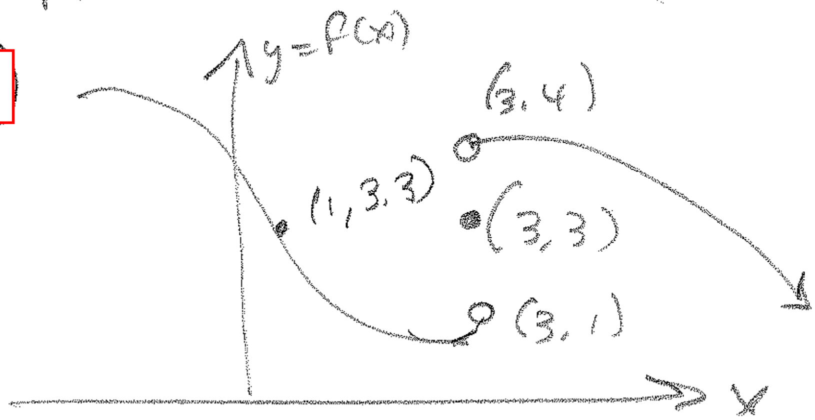
It's <u>entirely</u> possible $f(2) = 3$. (Just not nearby!)

2 Explain the meaning of...

$\lim_{x \to 1^-} f(x) = 3$ & $\lim_{x \to 1^+} f(x) = 7$

This says limit from the left as x approaches 1 is 3 and limit from the right as x approaches 1 is 7.

3



Find each of the following. If one doesn't exist, explain why.

(a) $\lim_{x \to 1} f(x) = 3.3$ (approx)

(b) $\lim_{x \to 3^-} f(x) = 1$

(c) $\lim_{x \to 3^+} f(x) = 4$

(d) $\lim_{x \to 3} f(x) \nexists$, b/c left- & right-hand limits don't agree.

(e) $f(3) = 3$

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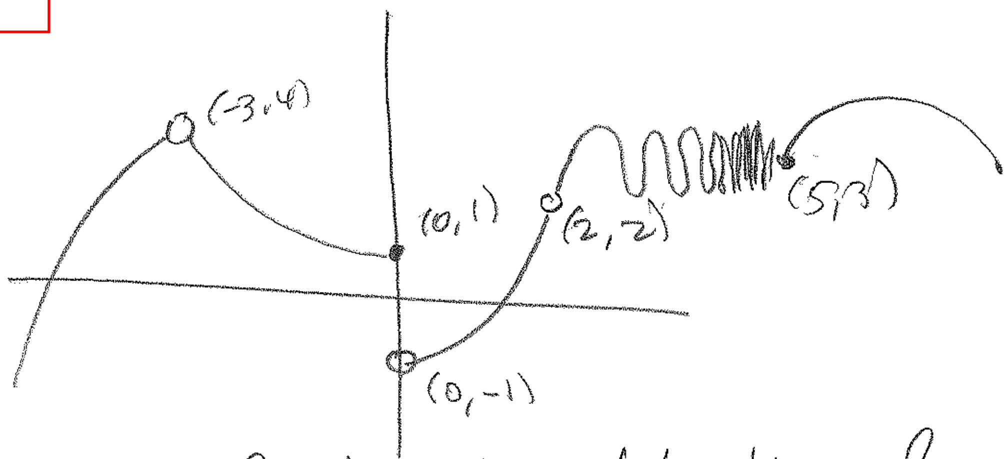
4



For this function, h, state the value, if it exists. If it doesn't, state why.

(a) $\lim_{x \to -3^-} h(x) = 4$

(b) $\lim_{x \to -3^+} h(x) = 4$

(c) $\lim_{x \to 3} h(x) = 4$

(d) $h(-3)$ $\nexists$. see open dot @ (-3,4) & no closed dot @ x=3.

(e) $\lim_{x \to 0^-} h(x) = 1$

(f) $\lim_{x \to 0^+} h(x) = -1$

(g) $\lim_{x \to 0} h(x)$ $\nexists$, b/c $\lim_{x \to 0^-} h(x) = 1 \neq -1 = \lim_{x \to 0^+} h(x)$.

(h) $h(0) = 1$

(i) $\lim_{x \to 2} h(x) = 2$

(j) $h(2)$ $\nexists$ No closed dot.

(k) $\lim_{x \to 5^+} h(x) = 3$

(l) $\lim_{x \to 5^-} h(x)$ $\nexists$. oscillates from 2 to 4 infinitely often in the neighborhood of x=5.

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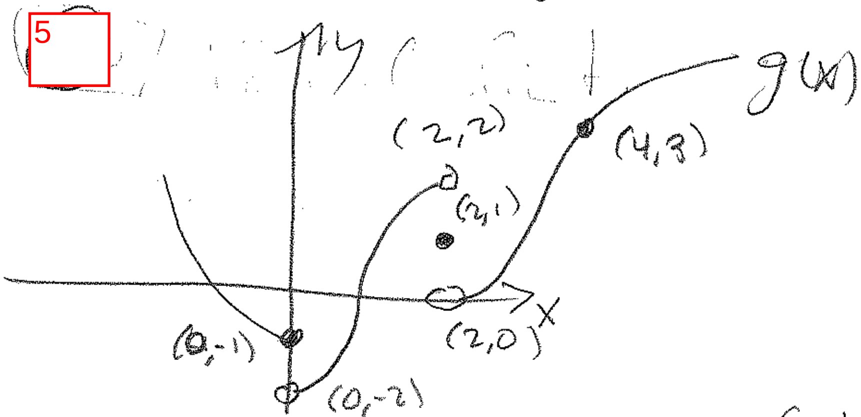
5



Same as #6

(a) $\lim_{x\to 0^-} g(x) = -1$

(b) $\lim_{x\to 0^+} g(x) = -2$

(c) $\lim_{x\to 0} g(x) \nexists$

left- & right-hand limits disagree.

(d) $\lim_{x\to 2^-} g(x) = 2$

(e) $\lim_{x\to 2^+} g(x) = 0$

(f) $\lim_{x\to 2} g(x) \nexists$

left-and right-hand limits disagree.

(g) $g(2) = 1$

(h) $\lim_{x\to 4} g(x) = 3$

sketch the graph and determine all $a \ni \lim_{x\to a} f(x)$ exists.

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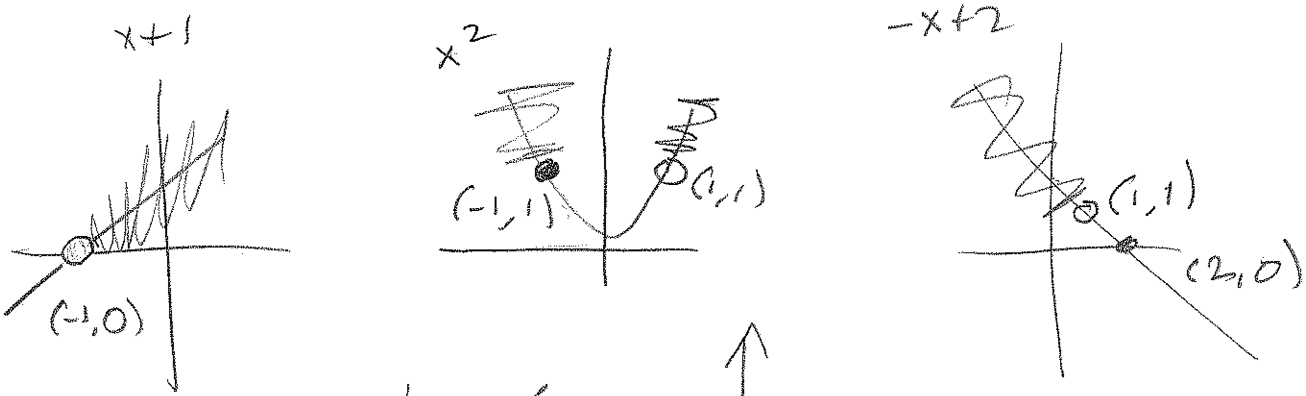
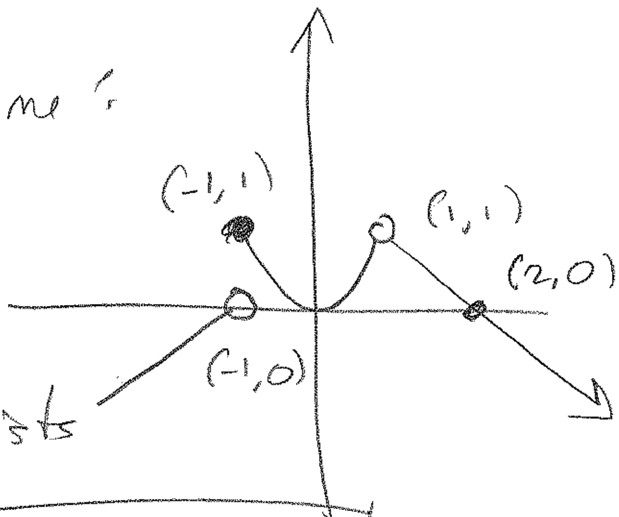
6

$$f(x) = \begin{cases} 1+x & x < -1 \\ x^2 & -1 \le x < 1 \\ 2-x & x \ge 1 \end{cases}$$



Combine:

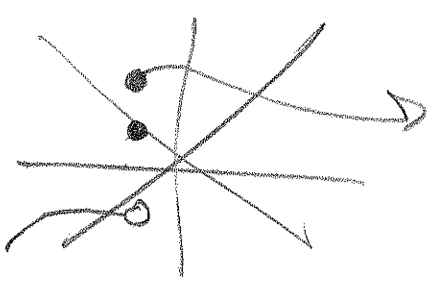


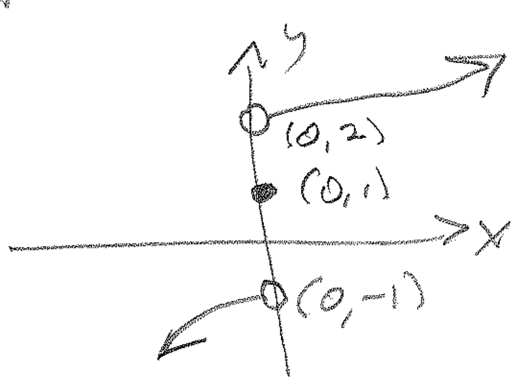
$\lim_{x \to a} f(x)$ exists

for $\boxed{a \in (-\infty,-1) \cup (-1,\infty)}$

Sketch a function that satisfies the given properties

7 $\lim_{x \to 0^-} f(x) = -1$ $\lim_{x \to 0^+} f(x) = 2$, $f(0) = 1$





201 S'15

Guess the limit by numerical method

8 $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{x^2 - x - 2} = \lim\limits_{x \to 2} f(x)$

$f(2.001) \approx .66678$ $\quad$ $f(1.9999) \approx .6666556$

$f(2.0001) \approx .666678$

$f(2.000001) \approx .66666678$

I'd guess $\lim\limits_{x \to 2} f(x) = .\overline{6} = \dfrac{2}{3}$.

Analytic Check:

$$\frac{x^2 - 2x}{x^2 - x - 2} = \frac{x(x-2)}{(x+1)(x-2)} = \frac{x}{x+1} \xrightarrow{x \to 2} \frac{2}{2+1} = \frac{2}{3}$$

$(x \neq 2)$

Use numerical methods

9 $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x} = \lim\limits_{x \to 0} f(x)$

My guess: $\boxed{\lim\limits_{x \to 0} f(x) = .25}$

$f(.001) \approx .24998$

$f(-.001) \approx .25002$

Analytic Check:

$$\left(\frac{\sqrt{x+4} - 2}{x}\right)\left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}\right) = \frac{x + 4 - 4}{x(\sqrt{x+4} + 2)} = \frac{1}{\sqrt{x+4} + 2}$$

$$\xrightarrow{x \to 0} \frac{1}{\sqrt{0+4} + 2} = \frac{1}{2+2} = \frac{1}{4} = .25$$

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10 $\displaystyle\lim_{x\to 1}\frac{x^6-1}{x^{10}-1}=\lim_{x\to 1}f(x)$

$f(.9999)\approx .60012$

$f(1.0001)\approx .59989$

My Guess:

$$\boxed{\lim_{x\to 1}f(x)=.6}$$

Analytic Check:

$$\frac{x^6-1}{x^{10}-1}=\frac{(x-1)(x^5+x^4+x^3+x^2+x+1)}{(x-1)(x^9+x^8+\cdots+x+1)}$$

$$=\frac{x^5+x^4+x^3+x^2+x+1}{x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1}\xrightarrow{x\to 1}\frac{6}{10}=.6\ \checkmark$$

11 (a) Graph & zoom to estimate

$$\lim_{x\to 0}\frac{\cos(2x)-\cos x}{x^2}=\lim_{x\to 0}\frac{\cos^2x-\sin^2x-\cos x}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{\cos x(\cos x-1)}{x^2}-\frac{\sin^2 x}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{\cos x-1}{x}\cdot\frac{\cos x}{x}-\left(\frac{\sin x}{x}\right)^2\right)$$

$\to 1$ $\qquad \to \infty$ $\qquad \to 1$

I don't think it exists

<u>But</u> my graph seems to think the $\displaystyle\lim_{x\to 0}f(x)\approx -.5$. My analysis isn't helping much. Numerically, $\boxed{y=L=-.5}$

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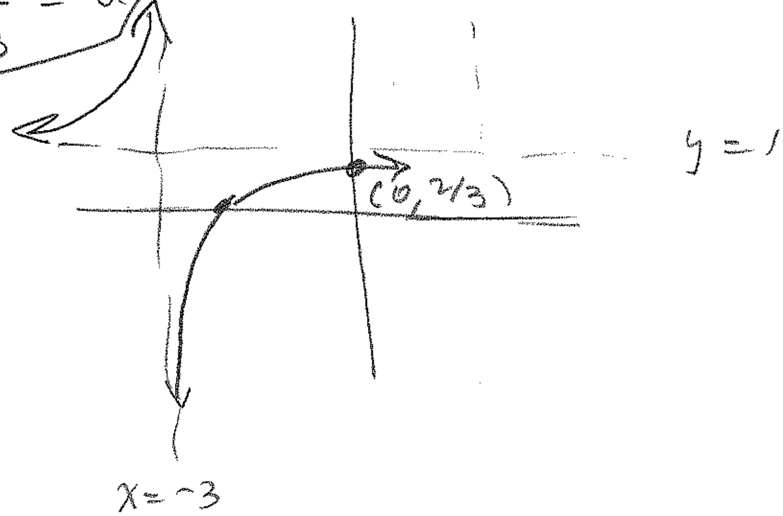
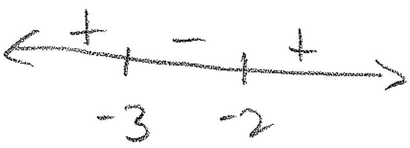
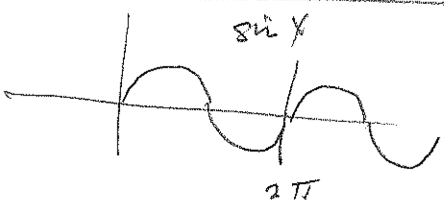
Determine the infinite limit ~~#s~~ 29–37

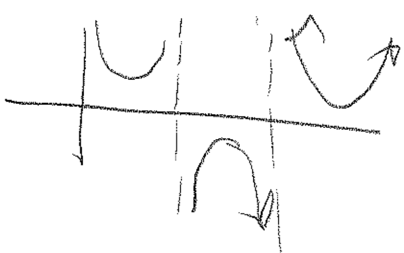
12. $$\boxed{\lim_{x \to -3^+} \frac{x+2}{x+3} = -\infty}$$





13. $$\boxed{\lim_{x \to 2\pi^-} x \csc x = -\infty}$$





x csc x won't be much different, qualitatively. Just $x \approx 2\pi$ times graph of csc x.

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14 (a) Find vertical asymptotes. (b) Confirm by graphing

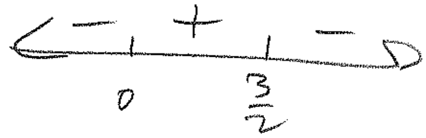
(a) $y = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$

$\boxed{x=0}$ $\quad 3-2x=0$

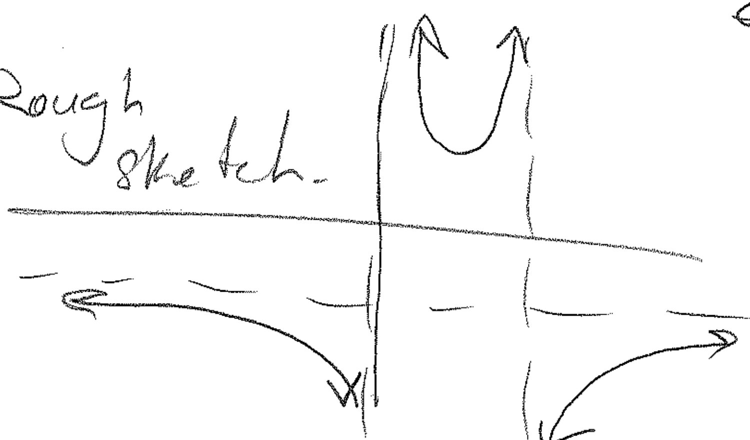
$2x=3$

$\boxed{x=\frac{3}{2}}$ → Vertical asymptotes

$\lim\limits_{x\to\infty} f(x) = \dfrac{1}{-2} = -\dfrac{1}{2} = y$



Rough sketch.



$y = -\frac{1}{2}$

$x=0 \qquad x=\frac{3}{2}$