

201 §1.4

1

The most useful one of these is the LAST one, beginning @ $t=2$, ending $t=2.001$. Left alone, I'd just plug in 2.001,
 $(t_1, y_1) = (2, 16)$

$$y(2) = -16(2)^2 + 40(2) = -16(4) + 80 = -64 + 80 = 16$$

$$(t_2, y_2) = (2.001,$$

$$f(t) = -16t^2 + 40t$$

$$f(2.001) = -16(2.001)^2 + 40(2.001)$$

$$(2 + .001)^2 = 2^2 + 2(2)(.001) + (.001)^2$$

$$= 4 + .004 + .000001 = 4.004001$$

$$(.001)^2 = (10^{-3})^2 = 10^{-6} = .000001.$$

$$\rightarrow = -16(4.004001) + 40(2.001)$$

$$= -64.064016 + 80.04 = 15.975984$$

$$\frac{y_2 - y_1}{t_2 - t_1} = \frac{15.975984 - 16}{2.001 - 2} = \frac{-0.024016}{.001}$$

$$= \boxed{-24.016}$$

My guess is it's

trending to $\boxed{-24}$.

Check $f'(t) = -32t + 40 \rightarrow$
 $f'(2) = -32(2) + 40 = -64 + 40 = -24$ is exact

201 § 1.4

1

This is one where it's easy to be deceived, especially (a) $x=0$. But it's still oscillating rapidly enough at $x=1$ for $x = 1.5, 1.4, 1.3$, and so forth, to jump around.

IF the limit exists (a) $x_1=1$, then START with $x_2 = 1.001$ or $x_2 = 0.999$, and come from right from left at it from both directions.

(a) They DON'T appear to approach a limit.

(b) We modeled this in class, showing how far apart the secant lines were for the (stupid) choices given to us.

(c) Try $x = 2.0001$:

$$\frac{f(1.0001) - f(1)}{.0001} \approx 31.41905675$$

$$\frac{f(0.9999) - f(1)}{-.0001} \approx \dots$$

$$x_2 = 1.999999 \Rightarrow \frac{f(x_2) - f(x_1)}{x_2 - x_1} \approx 31.41592654$$

As what I'm guessing is the limit of the slope of the secant line.

$$\dots \approx 31.41592654$$

I never did quite finish this 1.4 #2 problem. Let's do some computer algebra and see what we have here:

Define the function.

$$f := x \rightarrow \sin\left(\frac{10 \cdot \text{Pi}}{x}\right)$$

$$x \rightarrow \sin\left(\frac{10 \pi}{x}\right) \quad (1)$$

Define the slope of the secant line ("ss" for "secant slope," below:)

$$ss := x \rightarrow \frac{(f(x) - f(1))}{x - 1}$$

$$x \rightarrow \frac{f(x) - f(1)}{x - 1} \quad (2)$$

So the secant slope function looks like this :

$$ss(x)$$

$$\frac{\sin\left(\frac{10 \pi}{x}\right)}{x - 1} \quad (3)$$

Computer algebra system can do the limit, directly, which is nice. At this point, all we have is a numerical sledgehammer to find the limit:

$$\text{limit}(ss(x), x = 1)$$

$$-10 \pi \quad (4)$$

No way the numerical approach gives you a nice, symbolic -10π . But if you're persistent, enough, you'd come pretty darn close to the following digital answer:

And this last, digital answer is the answer to the final part of #2 that got cut off the bottom of the solutions page.

`evalf(%)`

`-31.41592654`

(5)