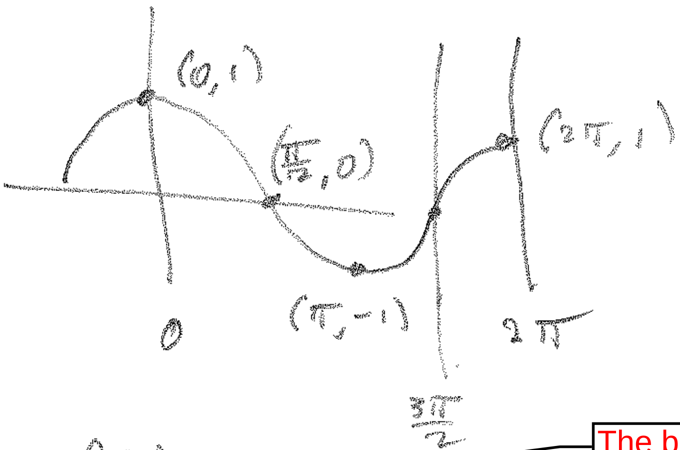
201 §1.3

Graph the func. by hand by transforming a standard func.

1 $y = \frac{1}{2}(1 - \cos x) = \frac{1}{2} - \frac{1}{2}\cos x = -\frac{1}{2}\cos x + \frac{1}{2}$



$f(x) = \cos x$

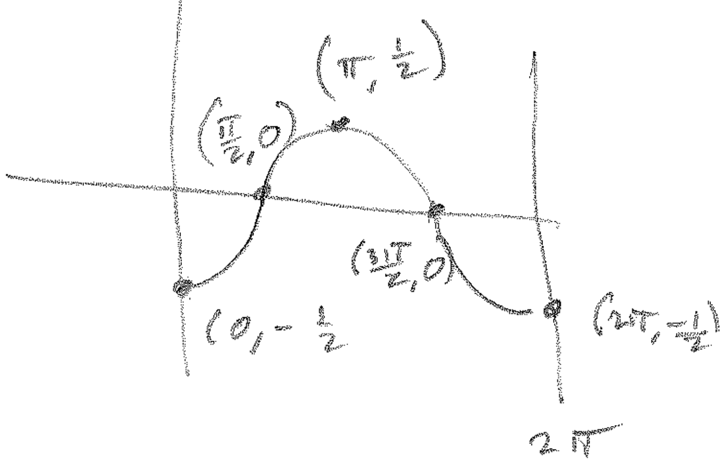
The basic function is f(x) = cos(x)

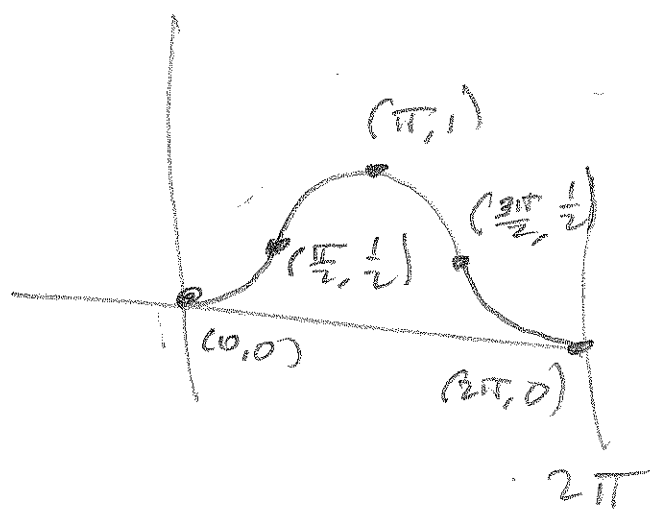
$-\frac{1}{2}f(x) = -\frac{1}{2}\cos x$

$(x, y) \longmapsto (x, -\frac{1}{2}y)$

$y = -\frac{1}{2}\cos x + \frac{1}{2} = -\frac{1}{2}f(x) + \frac{1}{2}$

$(x, y) \longmapsto (x, y + \frac{1}{2})$





201 §1.3

2 $y = 1 - 2x - x^2$

$f(x) = -x^2 - 2x + 1$

$-f(x) = x^2 + 2x - 1$

$-f(x) + 1 = x^2 + 2x$

$-f(x) + 1 + 1^2 = x^2 + 2x + 1^2$

$-f(x) + 2 = (x+1)^2$

$-f(x) = (x+1)^2 - 2$

$f(x) = -(x+1)^2 + 2$

$f(x) \overset{\text{SET}}{=} 0$

$-(x+1)^2 + 2 = 0$

$(x+1)^2 = 2$

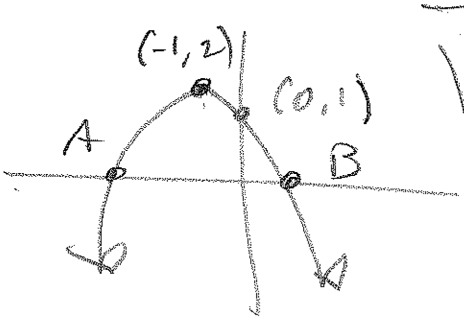
$(x+1) = \pm\sqrt{2}$

$x = -1 \pm \sqrt{2}$

$x^2 \longrightarrow -x^2 \longrightarrow -(x+1)^2 \longrightarrow -(x+1)^2 + 2$

flip vertically
left 1
up 2

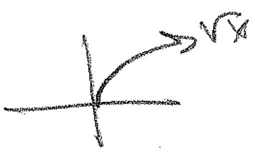
$(h, k) = (-1, 2)$
opens down

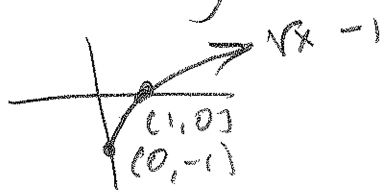


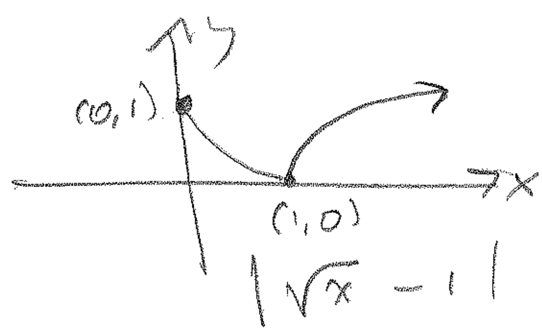
$$\boxed{\begin{aligned} A &= (-1-\sqrt{2}, 0) \\ B &= (-1+\sqrt{2}, 0) \end{aligned}}$$

3 $y = |\sqrt{x} - 1|$

I'd attack it by doing $\sqrt{x} \longrightarrow \sqrt{x} - 1 \longrightarrow |\sqrt{x} - 1|$



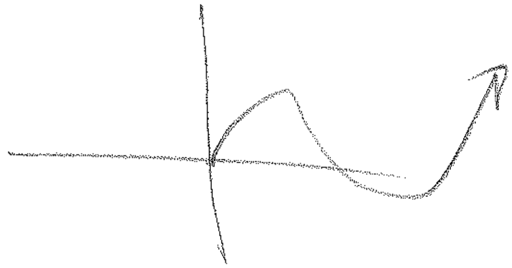




201 §1.3:

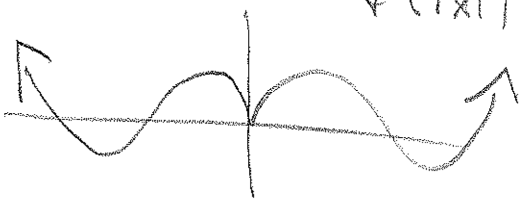
(a) The graph of $f(|x|)$ is obtained by reflecting the graph of $f(x)$ for $x \geq 0$ thru the y-axis

$f(x)$ on the $[0, \infty)$ domain.



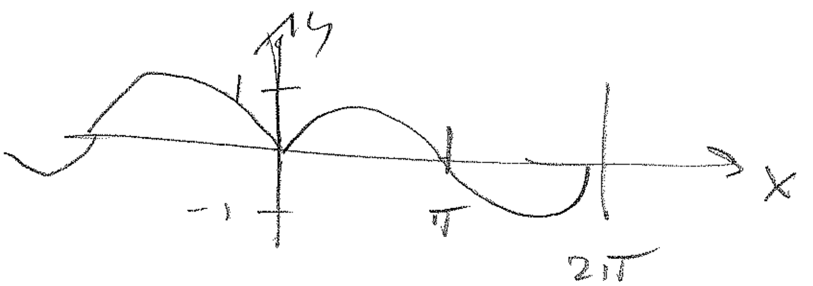
Now this is:

$f(|x|)$

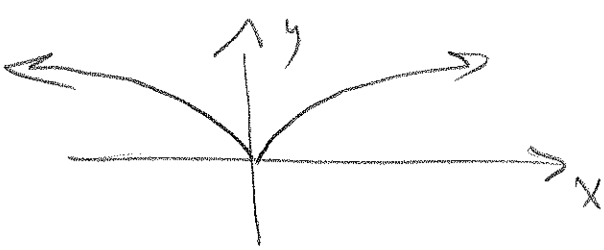


This function is EVEN.

(b) $f(x) = \sin x \Longrightarrow f(|x|) = \sin(|x|)$



(c) $f(x) = \sqrt{x} \longrightarrow f(|x|) = \sqrt{|x|}$



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$f(x) = x^3 + 2x^2,\ g(x) = 3x^2 - 1 \longrightarrow$

(a) $\boxed{(f+g)(x) = x^3 + 2x^2 + 3x^2 - 1 \quad \text{or } x^3 + 5x^2 - 1}$ $\quad \mathcal{D} = \mathbb{R}$

(b) $\boxed{(f-g)(x) = x^3 + 2x^2 - 3x^2 + 1 = x^3 - x^2 + 1}$ $\quad \mathcal{D} = \mathbb{R}$

(c) $(fg)(x) = (x^3 + 2x^2)(3x^2 - 1)$

$= 3x^5 - x^3 + 6x^4 - 2x^2$

$= 3x^5 + 6x^4 - x^3 - 2x^2$ $\quad \mathcal{D} = \mathbb{R}$

(d) $\boxed{\left(\frac{f}{g}\right)(x) = \dfrac{x^3 + 2x^2}{3x^2 - 1}}$

$\mathcal{D} = \{x \mid 3x^2 - 1 \neq 0\} = \{x \mid x \neq \pm\sqrt{\tfrac{1}{3}}\}$

$= \{x \mid x \neq \pm \frac{\sqrt{3}}{3}\} = (-\infty, -\frac{\sqrt{3}}{3}) \cup (-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

Domain

2'01 S^n 1.3

Find (a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ & (d) $g \circ g$

6

$f(x) = x^2 - 1,\ g(x) = 2x + 1 \qquad \mathcal{D}(f) = \mathbb{R} = \mathcal{D}(g)$

(a) $(f \circ g)(x) = f(g(x)) = (g(x))^2 - 1 = \boxed{(2x+1)^2 - 1 = (f \circ g)(x)}$

$\mathcal{D} = \mathbb{R}$

(b) $\boxed{(g \circ f)(x) = 2(x^2 - 1) + 1} \qquad \mathcal{D} = \mathbb{R}$

(c) $\boxed{(f \circ f)(x) = (x^2 - 1)^2 - 1} \qquad \mathcal{D} = \mathbb{R}$

(d) $\boxed{(g \circ g)(x) = 2(2x+1) + 1} \qquad \mathcal{D} = \mathbb{R}$

7

$f(x) = x + \frac{1}{x},\ g(x) = \frac{x+1}{x+2} \qquad \mathcal{D}(f) = \mathbb{R} \setminus \{0\}, \ \mathcal{D}(g) = \mathbb{R} \setminus \{-2\}$

(a) $(f \circ g)(x) = \dfrac{x+1}{x+2} + \dfrac{1}{\frac{x+1}{x+2}} = \dfrac{x+1}{x+2} + \dfrac{x+2}{x+1}$

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq -2 \text{ AND } \frac{x+1}{x+2} \neq 0\}$ $\rightarrow g(x) \in \mathcal{D}(f)$!

$= \{x \mid x \neq -2 \text{ AND } x \neq -1\}$

$= (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

$\frac{x+1}{x+2} = 0$

$x + 1 = 0$

$x = -1$

201 §1.3

7 cnt'd

(b) $(g \circ f)(x) = \dfrac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2}$

$\mathcal{D}(g \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ AND } f(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \neq 0 \text{ and } x + \frac{1}{x} \neq -2\}$

$= \{x \mid x \neq 0 \text{ and } x \neq 1\} = (-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$x + \frac{1}{x} = 2$

$\dfrac{x^2 + 1}{x} = \dfrac{2x}{x}$

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

(c) $(f \circ f)(x) = x + \frac{1}{x} + \dfrac{1}{x + \frac{1}{x}} = \dfrac{x^2+1}{x} + \dfrac{1}{\frac{x^2+1}{x}}$

$= \dfrac{x^2+1}{x} + \dfrac{x}{x^2+1}$

$= \dfrac{(x^2+1)^2 + x^2}{x(x^2+1)}$ so? who cares?

!

$\mathcal{D}(f \circ f) = \{x \mid x \in \mathcal{D}(f) \text{ and } f(x) \in \mathcal{D}(f)\}$

$= \{x \mid x \neq 0 \text{ and } x + \frac{1}{x} \neq 0\}$

$= \{x \mid x \neq 0\}$

201 §1.3

7c cnt'd

Scratch: $x + \frac{1}{x} = 0$

$\frac{x^2+1}{x} = 0$

$x^2+1 = 0$ No real sol'n, so

$x + \frac{1}{x} \in \mathcal{D}(f)$ is no restriction

(d) $\boxed{(g \circ g)(x) = \dfrac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2}}$

$\mathcal{D}(g \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(g)\}$

$= \{x \mid x \neq -2 \text{ AND } g(x) \neq -2\}$

$= \{x \mid x \neq -2 \text{ AND } x \neq -\frac{5}{3}\}$

$=$

$= (-\infty, -2) \cup (-2, -\frac{5}{3}) \cup (-\frac{5}{3}, \infty)$

Scratch: $\frac{x+1}{x+2} = -2$

$x+1 = -2x-4$

$3x = -5$

$x = -\frac{5}{3}$

201 §1.3

Express as a composition $f \circ g$

$$F(x) = \frac{\sqrt[3]{x}}{\sqrt[3]{x} + 1}$$

$$\boxed{\text{Let } g(x) = \sqrt[3]{x} \text{ and } f(x) = \frac{x}{x+1}. \text{ Then } F(x) = (f \circ g)(x)}$$

Pain in the neck picture goes here:

Use graph to evaluate or explain why it can't.

(a) $f(g(2)) = f(5) = 4$

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$ $\nexists$ b/c $6 \notin \mathcal{D}(g)$.

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

201 S[?] 1.3

10

A spherical balloon is being inflated. Its radius is increasing at a (constant) rate of 2 cm/s.

(a) Express the radius r as a function of time t, in seconds: $r = 2t$

(b) Find V = Volume as function of r:

I'm not quite sure what they mean.

I know $V(r) = \frac{4}{3}\pi r^3$ and I know $r(t) = 2t$, so that Volume as function of time is $V(r(t)) = \frac{4}{3}\pi (2t)^3 = \frac{32}{3}\pi t^3$.

This is unusual model. Usually volume changes at a constant rate and radius grows more slowly as volume increases.