MAT 201 Section 3.2 questions 3/4/15

#s 1 – 3: Verify that the hypotheses of Rolle's Theorem (Cnt$^{\underline{s}}$ on closed interval and Difb$^{\underline{l}}$ on open interval) are satisfied and then find the c that Rolle's says will be there. If the hypo's are *not* satisfied, then you're done, by explaining why it doesn't satisfy Rolle's. Also, there may be more than one c.

1. $f(x) = 3x^2 - 12x + 5$ on $[1,3]$.
2. $f(x) = \sqrt{x} - \frac{1}{3}x$ on $[0,9]$.
3. $f(x) = 1 - x^{\frac{2}{3}}$ on $[-1,1]$.

#s 4, 5 : Same instructions, only this time, it's the Mean Value Theorem, which loosens up the "must be equal" condition at the endpoints. MVT is really just a generalization of Rolle's.

4. $f(x) = 2x^2 - 3x + 1$ on $[0,2]$.
5. $f(x) = \sqrt[3]{x}$ on $[0,1]$.
6. Show that $(x-3)^{-2}$ does *not* yield a c in $(1,4)$ such that $f'(c) = m_{avg} = \dfrac{f(b) - f(a)}{b-a}$ on the interval.

   Why does this *not* violate the Mean Value Theorem?

   An alternate statement of the MVT conclusion is "… c such that $f'(c)(b-a) = f(b) - f(a)$." This alternate amounts to just multiplying both sides of the original by $(b-a)$. I think the first way relates to slopes and you can draw the picture. This second way is saying you can find the Net Change in well-behaved function by multiplying a particular value of the derivative by the width of the interval.

7. Show that $f(x) = 2x + \cos(x)$ has *exactly* one root in the interval $[-2,2]$.
8. Show that the equation $x^3 - 15x + c = 0$ has *exactly* one root, regardless of the value of c.
9. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \le 2$ for all x?
10. Show that $\sqrt{x+1} < 1 + \frac{1}{2}x$ if $x > 0$. (Hint: Consider the function $f(x) = \sqrt{x+1} - 1 - \frac{1}{2}x$. Analyze its derivative.)