Section 2.4 questions

2/13/15

#s 1 – 8: Differentiate:

1. $f(x) = 3x^2 - 2\cos x$ 2. $f(x) = \sin x + \frac{1}{2}\cot x$ 3. $y = \sec \theta \tan \theta$ 4. $g(t) = \cos t + t^2 \sin t$ 5. $y = \frac{x}{2 - \tan x}$ 6. $f(\theta) = \frac{\sec \theta}{1 + \sec \theta}$ 7. $y = \frac{t \sin t}{1 + t}$

#s 9, 10:Prove the following results, using the derivatives of sine and cosine as given (as in class).

9.
$$\frac{d}{dx}[\csc x]$$
 10. $\frac{d}{dx}[\cot x]$

#s 11, 12: Find an equation of the tangent line to the given curve at the given point.

- 11. $y = \sec x$, $(x_1, y_1) = \left(\frac{\pi}{3}, 2\right)$ 12. $y = \cos x - \sin x$, $(x_1, y_1) = (\pi, 1)$
- 13. Consider the function $f(x) = 2x \sin x$.

a. Find an equation of the tangent line to f at the point $P\left(\frac{\pi}{2},\pi\right)$

b. Sketch a graph illustrating what you just did. Use a grapher to assist you.

- 14. Let $f(x) = \sec x x$.
 - a. Find f'(x).
 - b. Use a graphing utility to see if your work makes sense, by graphing f and f' together on the same graph.
- 15. Find the first two derivatives of $H(\theta) = \theta \sin \theta$
- 16. Differentiate $f(x) = \frac{\tan x 1}{\sec x}$ in two ways:
 - a. By quotient rule on f as it appears.
 - b. By rewriting tangent and secant in terms of cosines and sines, simplifying, and *then* taking the derivative.
 - c. See if you can make both answers look the same! Equivalent trig expressions can look very very different from one another, and being able to experiment with how they look can make harder problems into easier problems. You just have to balance the time spent looking for better representations with the time spent just biting the bullet and following your nose to the next step on a possibly very messy expression.

- 17. For what values of x does $f(x) = x + 2\sin x$ have a horizontal tangent Love this question on a test!
- 18. A mass on a spring is vibrating on a smooth, horizontal surface. Its motion back and forth is described by the function $x(t) = 8 \sin t$, where t is time, in seconds, and x is its displacement from zero, in centimeters.
 - a. Find the velocity and acceleration functions.

b. Evaluate
$$x\left(\frac{2\pi}{3}\right)$$
, $x'\left(\frac{2\pi}{3}\right)$, and $x''\left(\frac{2\pi}{3}\right)$.

#s 19, 20: Evaluate the limit.

19.
$$\lim_{x \to 0} \frac{\sin(3x)}{x}$$

20.
$$\lim_{x \to 0} \frac{\sin(3x)}{5x^3 - 4x}$$

21.
$$\lim_{x \to \infty} \left(x \sin\left(\frac{1}{x}\right) \right)$$

22.
$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$