MAT 201 Section 1.6 questions 1/22/15

1. Given that

$$\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$$

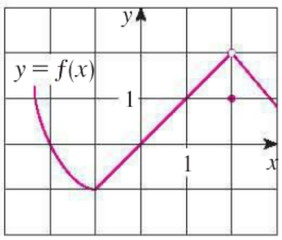
find the limits that exist. If the limit does not exist, explain why.

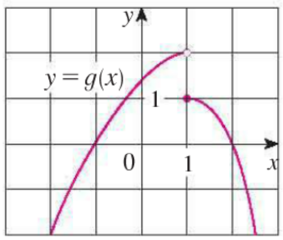
(a) $\lim_{x\to 2} [f(x) + 5g(x)]$ (b) $\lim_{x\to 2} [g(x)]^3$

(c) $\lim_{x\to 2} \sqrt{f(x)}$ (d) $\lim_{x\to 2} \dfrac{3f(x)}{g(x)}$

(e) $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$ (f) $\lim_{x\to 2} \dfrac{g(x)h(x)}{f(x)}$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





(a) $\lim_{x\to 2} [f(x) + g(x)]$ (b) $\lim_{x\to 1} [f(x) + g(x)]$

(c) $\lim_{x\to 0} [f(x)g(x)]$ (d) $\lim_{x\to -1} \dfrac{f(x)}{g(x)}$

(e) $\lim_{x\to 2} [x^3 f(x)]$ (f) $\lim_{x\to 1} \sqrt{3 + f(x)}$

#s 3, 4 Evaluate the limit and justify each step by using the appropriate limit laws. This sort of exercise is obligatory, although of relatively limited value, in my eyes. Your intuition is a great guide: You *want* to be able to move the limit operator over/through/across the various arithmetic operations, and you basically *can*. The thing to focus on before you do such a move, is whether the two parts (like the two members in a sum) *both* have limits, separately? If so, the "limit of the **sum** is the **sum** of the limits." In place of the word "sum," you can put "product, power, root, quotient, etc.

3. $\lim_{t\to -2} \dfrac{t^4 - 2}{2t^2 - 3t + 2}$

4. $\lim_{x\to 2} \sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

5. (a) What is wrong with the following equation?

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

(b) In view of part (a), explain why the equation

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x\to 2} (x + 3)$$

is correct.

#s 6 – 10 Evaluate the limit, if it exists. If it doesn't, explain why.

A careless student will factor this wrong.

6. $\lim_{x\to 5} \frac{x^2 - 6x + 5}{x - 5}$

7. $\lim_{x\to 5} \frac{x^2 - 5x + 6}{x - 5}$

8. $\lim_{x\to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

9. $\lim_{t\to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$

Rationalize the numerator!

10. $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

Tangent slope function for a cubic!

11. Prove that $\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0.$

Squeeze it!

#s 12 – 14 Find the limit, if it exists. If it doesn't exist, explain why.

12. $\lim_{x\to 0.5^-} \frac{2x - 1}{|2x^3 - x^2|}$

13. $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

14. $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

Requires a sign pattern on what's inside the absolute value!

15. Let $g(x) = \frac{x^2 + x - 6}{|x - 2|}$.

(a) Find

(i) $\lim_{x\to 2^+} g(x)$ (ii) $\lim_{x\to 2^-} g(x)$

(b) Does $\lim_{x\to 2} g(x)$ exist?

(c) Sketch the graph of g.

16. In the theory of relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v\to c^-} L$ and interpret the result. Why is a left-hand limit necessary?

Math modeling question

17. If $\lim_{x\to 0} \frac{f(x)}{x^2} = 5$, find the following limits.

(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 0} \frac{f(x)}{x}$

18. Is there a number a such that

$$\lim_{x\to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

exists? If so, find the value of a and the value of the limit.

So the denominator goes to zero as *x* approaches -2, right? Somehow, you need the same from the numerator, or the limit will be a division by zero.