

1. If a ball is thrown into the air with a velocity of 40 ft/s, its height, h , in feet, is given by $h(t) = 40t - 16t^2$, where t is time, in seconds.
 - a. Find the average velocity for the time period beginning when $t = 2$, and lasting
 - i. 0.5 seconds
 - ii. 0.1 seconds
 - iii. 0.05 seconds
 - iv. 0.01 seconds
 - b. Estimate the instantaneous velocity when $t = 2$.
2. The point $P(1,0)$ lies on the curve $f(x) = \sin\left(\frac{10\pi}{x}\right)$. Here, we explore what can happen, when the limit exists, but the function bounces around so much, it's hard to tell, until we get *really* close to the limiting value of $x = 1$.
 - a. Build a secant function $m(x) = \frac{f(x) - f(1)}{x - 1}$. Notice it is not defined at $x = 1$. We're exploring whether, like all smooth functions, it has a limit, as x approaches 1. If you have a graphing calculator, or access to a grapher, such as this one: <http://dlippman.imathas.com/graphcalc/graphcalc.html>, you can work part (b) rather quickly!
 - b. If Q is the point $Q(x, f(x))$, find the slope of the secant line PQ (correct to 4 decimal places) for $x = 2, 1.5, 1.4, 1.3, 1.2, 1.1, 0.5, 0.6, 0.7, 0.9$.
 - c. Use a grapher to render a graph of $f(x)$ and explain why the slopes of the secant lines in part (a) are not close to the slope of the tangent line at P .
 - d. By choosing *appropriate* values of x (sufficiently close to $x = 1$), you should be able to estimate the slope of the curve at $x = 1$.

#2, above, illustrates one of the subtleties of limits. Your intuition is *usually* an excellent guide, but we want rules that work for *all* functions and *all* situations. The function in #2 is a version of *The Topologist's Sine Curve*. Notice it is not defined at $x = 0$. It also oscillates infinitely often on any neighborhood of $x = 0$. It oscillates enough in the vicinity of $x = 1$ to cause us some technical difficulties, there, as well.