MAT 201, Test 4, Spring, 2011 100 Points

Name_____

1. (20 pts) Evaluate the definite integral
$$\int_{0}^{1} (6x^{5} - 6x^{2} + 1) dx$$

2. (10 pts) Evaluate the definite integral
$$\int_{0}^{\frac{\pi}{6}} (\cos(2t)) dt$$

3. (10 pts) Evaluate the indefinite integral $\int (4x^3 - 6x^2) \sqrt[3]{x^4 - 2x^3 + 1} dx.$

4. (10 points) Evaluate $\int_{0}^{3} (1 + x) dx$ by interpreting it in terms of areas of one or more

common geometric figures. In other words: Draw the picture! Find the area! You may use integration to *check* your answer, but I want to see that you know the picture and can figure the area by multiple means.

5. (10 pts) Find
$$g'(x)$$
, if $g(x) = \int_{0}^{2x^{3}} \frac{\sin(\pi t)}{t^{2} + 1} dt$, i.e., evaluate $\frac{d}{dx} \left[\int_{0}^{2x^{3}} \frac{\sin(\pi t)}{t^{2} + 1} dt \right]$

6. (5 pts) Expand
$$\left(5 + \frac{4k}{n}\right)^3$$

(**Bonus 5 pts**) Expand $(x-2)^5$.

Test 4

7. a. (10 pts) Estimate the area under $f(x) = 3x^2$ on the interval [0,4], using a Riemann sum, with 4 subintervals, and right endpoints. Supply a sketch that shows the graph of the function and the rectangles you use in your Riemann sum.

b. (10 pts) Use the definition of the definite integral to evaluate $\int_{0}^{4} 3x^2 dx$. This is the *long way*. Hint: $\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} = \frac{n^3 + \text{lower degree}}{3}$.

8. (10 pts) **Two versions of the same question:** The mountain trail from Ken's cabin takes a bee-line to Barbie's cabin over rough terrain. As the crow flies, the distance between the cabins is 4 miles. The steepness of the trail, measured in feet per mile, is described by the function $f(x) = 3x^2 - 6x + 2$. Whose cabin sits higher and by how much?

The slope (in feet per mile) of a trail is described by the function $f(x) = 3x^2 - 6x + 2$, where x measures the horizontal distance (map distance) from the trail head. What's the net change in altitude for a mountaineer who covers 4 miles (map distance) from the trail head?

(Bonus 5 pts) Evaluate $\int_{1}^{3} |x^3 - 3x^2 + 2x| dx$. Accompany your work with a sketch of the area(s) involved.