MAT 201, Test 3, Spring, 2011
100 Points

Name KEY

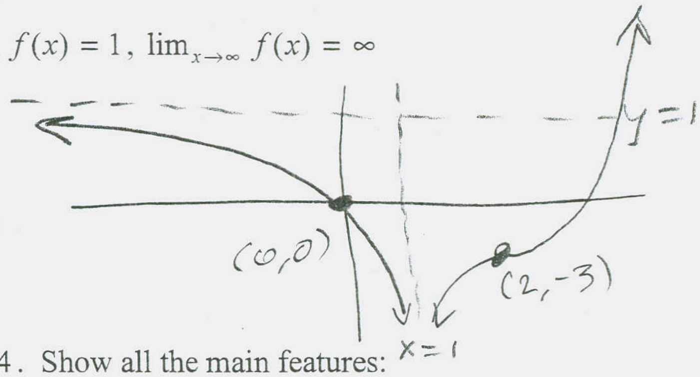
(20 pts) Answer one of the following versions of #1, below:

1. Use the information to sketch the graph of $f(x)$:

$\lim_{x\to 1^-} f(x) = -\infty$, $\lim_{x\to 1^+} f(x) = -\infty$, $\lim_{x\to -\infty} f(x) = 1$, $\lim_{x\to \infty} f(x) = \infty$

$f(0) = 0$, $f(2) = -3$, $f(3) = 0$

		1			
f'	neg	|	pos		
f"	neg	|	neg	|	pos
		1		2	



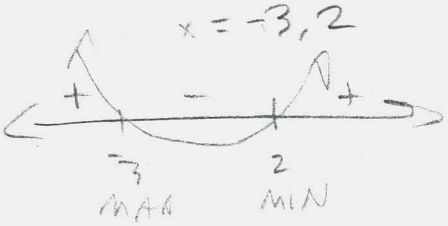
1. Sketch the graph of $f(x) = 2x^3 + 3x^2 - 36x - 54$. Show all the main features:
   a. x-intercept(s) ~~(Hint: There's one @ $x = 3$.)~~ xtra credit
   b. y-intercept(s)
   c. local extrema
   d. inflection point(s)

$f'(x) = 6x^2 + 6x - 36$

$= 6(x^2 + x - 6) \overset{SET}{=} 0$

$(x+3)(x-2) = 0 \rightarrow$

$x = -3, 2$



$f''(x) = 12x + 6 \overset{SET}{=} 0$

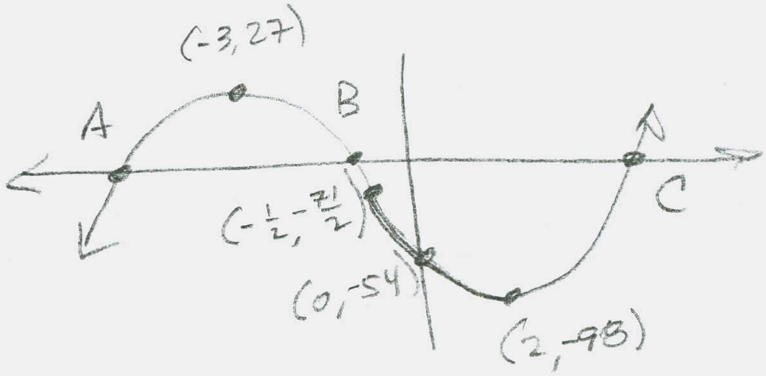
$x = -\frac{1}{2}$ IP

$2x^3 + 3x^2 - 36x - 54$

$= x^2(2x+3) - 18(2x+3)$

$= (x - 3\sqrt{2})(x + 3\sqrt{2})(2x+3)$

-3	2	3	-36	-54
		-6	9	81
	2	-3	-27	27



$-\frac{1}{2}$	2	3	-36	-54
		-1	-1	$\frac{37}{2}$
	2	2	-37	$-\frac{71}{2} = -35.5$

2	2	3	-36	-54
		4	14	-44
	2	7	-22	-98

$A = (-3\sqrt{2}, 0) \approx (-4.24, 0)$

$B = (-\frac{3}{2}, 0) = (-1.5, 0)$

$C = (3\sqrt{2}, 0) \approx (4.24, 0)$

2. (20 pts) Find the local and absolute extreme values of $f(x) = 2x^3 + 3x^2 - 36x - 54$ on [0,6].

$f(0) = -54$

$f(6) = 54$

$$\begin{array}{r|rrrr} 6 & 2 & 3 & -36 & -54 \\ & & 12 & 90 & 108 \\ \hline & 2 & 15 & 54 & 54 \end{array}$$

$f'(x) = 6x^2 + 6x - 36 \overset{\text{SET}}{=} 0$

$= 6(x^2 + x - 6) = 0$

$\Rightarrow (x+3)(x-2) = 0$

$\Rightarrow x = 2 \in [0,6]$

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -36 & -54 \\ & & 4 & 14 & -44 \\ \hline & 2 & 7 & -22 & -98 \end{array}$$

$\boxed{f(2) = -98 \text{ local \& Absolute Min}}$

$\boxed{f(6) = 54 \text{ Absolute Max}}$

3. (10 pts) Find all values c that satisfy the conclusion of the Mean Value Theorem for $f(x) = x^3 - 6x^2 + 5x + 6$ on $[0,6]$. For partial or extra credit, confirm that the hypotheses of the Mean Value Theorem are satisfied.

$f(0) = 6$

$f(6) = 36$

$$\begin{array}{r|rrrr} 6 & 1 & -6 & 5 & 6 \\ & & 6 & 0 & 30 \\ \hline & 1 & 0 & 5 & 36 \end{array}$$

$\frac{36-6}{6-0} = \frac{30}{6} = 5 = m$

$f'(x) = 3x^2 - 12x + 5 \overset{\text{SET}}{=} 5$

$3x(x-4) = 0$

$x = 0$ OR $x = \boxed{4 = c}$

Not in the interior

4. Find the limit.

a. (10 pts) $\lim_{t\to\infty} \frac{3t^3 - 4t^2 + 11}{5 - 3t - 12t^3}$ $= \frac{3}{-12} = -\frac{1}{4}$

b. (10 pts) $\lim_{t\to-\infty} \left(\sqrt{16t^2 - 7t} + 4t\right)$ $\left(\frac{\sqrt{16t^2-7t} - 4t}{\sqrt{16t^2-7t} - 4t}\right)$

$= \lim_{t\to-\infty} \frac{16t^2 - 7t - 16t^2}{\sqrt{16t^2-7t} - 4t} = \lim_{t\to-\infty} \frac{-7t}{\sqrt{t^2(16 - \frac{7}{t})} - 4t}$

$= \lim_{t\to-\infty} \frac{-7t}{-t\sqrt{16 - 7/t} - 4t} = \lim_{t\to-\infty} \frac{-t(7)}{-t(\sqrt{16 - 7/t} + 4)}$

$= \frac{7}{\sqrt{16} + 4} = \boxed{\frac{7}{8}}$

5. (10 pts) Given $f'(x) = \sec^2(x) - \sin(x)$ and $f(0) = 3$, find $f(x)$.

$f(x) = \tan(x) + \cos(x) + C$

$f(0) = 0 + 1 + C = 3$

$C = 2$

$\therefore \boxed{f(x) = \tan(x) + \cos(x) + 2}$

6. (10 pts) Find the point that minimizes the distance between the graph of the line $y = 3x + 1$ and the point $(4,3)$.

Distance is minimized when its square is

$D = (x-4)^2 + (3x+1-3)^2$ = square of distance.

It's a parabola that opens up.

METHOD 1:

$x^2 - 8x + 16 + (3x-2)^2$

$= x^2 - 8x + 16 + 9x^2 - 12x + 4$

$= 10x^2 - 20x + 20 = D \Rightarrow$

$D'(x) = 20x - 20 \overset{\text{SET}}{=} 0$

$\Rightarrow x = 1$

$\Rightarrow y = 3x + 1 = 4$

$\Rightarrow \boxed{(1,4)}$

METHOD 2

$D = (x-4)^2 + (3x-2)^2$

$\Rightarrow D' = 2(x-4) + 2(3x-2)(3)$

$= 2x - 8 + 6(3x-2)$

$= 2x - 8 + 18x - 12$

$= 20x - 20 \overset{\text{SET}}{=} 0$, etc.

7. (10 pts) Use Newton's method to find the second and third approximations to the real root of the equation $x^2 = 7$. For uniformity, use $x_1 = 2$, and approximate x_2 and x_3 to four decimal places.

$f(x) = x^2 - 7$

$f'(x) = 2x$

$$x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^2 - 7}{2x_k} = x_k - \frac{x_k}{2} + \frac{7}{2x_k} = \frac{x_k}{2} + \frac{7}{2x_k}$$

$$x_2 = \frac{2}{2} + \frac{7}{2(2)} = 1 + \frac{7}{4} = \frac{4+7}{4} = \frac{11}{4} = \boxed{2.750} \approx x_2$$

$$x_3 = \frac{2.75}{2} + \frac{7}{2(2.75)} = \frac{233}{88} \approx \boxed{2.6477} \approx x_3$$