MAT 201 Test 2
Spring, 2011 Name KEY
100 Points

1. (15 pts) Find $f'(x)$ by the definition of the derivative (The long way!) for
$f(x) = 2x^2 - 5x - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 5(x+h) - 1 - [2x^2 - 5x - 1]}{h}$$

$$= \frac{2(x^2+2xh+h^2) - 5x - 5h - 1 - 2x^2 + 5x + 1}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 5x - 5h - 1 - 2x^2 + 5x + 1}{h}$$

$$= \frac{4xh + 2h^2 - 5h}{h}$$

$$= \frac{h(4x + 2h - 5)}{h} \xrightarrow{h \to 0} \boxed{4x - 5 = f'(x)}$$

2. Find the first derivatives (5 pts each). Do not simplify!

a. $f(x) = x^2 - x^{-3} + 2\sqrt[3]{x} + \dfrac{2}{\sqrt{x}} + 111.234$

$$= x^2 - x^{-3} + 2x^{1/3} + 2x^{-\frac{1}{2}} + 111.234$$

$$\Rightarrow \boxed{f'(x) = 2x + 3x^{-4} + \frac{2}{3}x^{-\frac{2}{3}} - x^{-\frac{3}{2}}}$$

b. $g(x) = \dfrac{6x^5 + 2x^3 - 5x}{x^3 - 1}$ $\Rightarrow$

$$\boxed{g'(x) = \frac{(30x^4 + 6x^2 - 5)(x^3 - 1) - (6x^5 + 2x^3 - 5x)(3x^2)}{(x^3 - 1)^2}}$$

c. $h(x) = \sin^2\left(x^2 + \cos(x)\right)$

$$\Rightarrow \boxed{h'(x) = \left(2\sin\left(x^2 + \cos(x)\right)\right)\left(\cos\left(x^2 + \cos(x)\right)\right)\left(2x - \sin(x)\right)}$$

d. $\left(x^2 - 7x\right)\sin(2x) = y$

$$\Rightarrow \boxed{\frac{dy}{dx} = (2x-7)\sin(2x) + (x^2 - 7x)(\cos(2x))(2)}$$

3. (10 pts) Find an equation of the tangent line to $f(x) = \sqrt[3]{x^2}$ at the point $P = (1,1)$.

$$f(x) = x^{\frac{2}{3}} \Rightarrow f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$$

$$\Rightarrow f'(1) = \frac{2}{3}(1)^{-\frac{1}{3}} = \frac{2}{3} = m$$

$$y = m(x - x_1) + y_1$$

$$\boxed{y = \frac{2}{3}(x-1) + 1}$$

$$= \frac{2}{3}x - \frac{2}{3} + 1 \Rightarrow \boxed{y = \frac{2}{3}x + \frac{1}{3}}$$

4. (5 pts) Estimate $\sqrt[3]{(1.1)^2}$ using the Linearization of a particular function f at a handy value of x.

$a = 1$, $f(x) = \sqrt[3]{x^2}$, Cool. The work is done

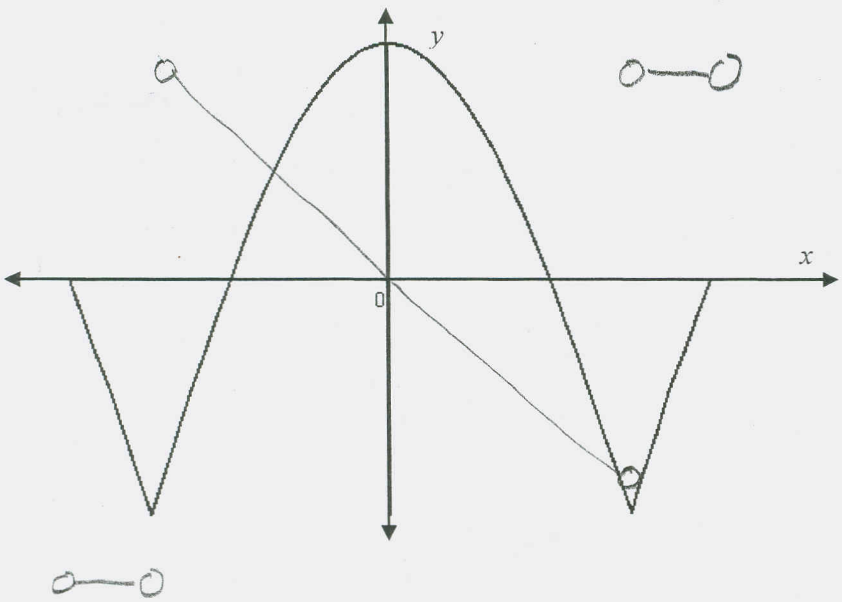
$$f(x) \approx L_a(x) = \frac{2}{3}(x-1) + 1 \Rightarrow$$

$$f(1.1) \approx \frac{2}{3}(1.1 - 1) + 1$$

$$= \frac{2}{3}(.1) + 1$$

$$= \frac{2}{30} + 1 = \frac{32}{30} = \boxed{1.0\overline{6} \approx \sqrt[3]{(1.1)^2}}$$

5. (10 pts) The graph of a function f is shown. Sketch the graph of f' on the same set of axes.



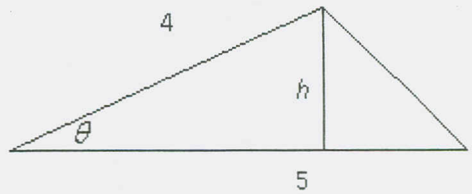
6. (15 pts) Find $\frac{dy}{dx}$, given $x^2y^3 - 5x^2 - 5y^2 = y^3 + 11.3$

$$\Rightarrow 2xy^3 + x^2(3y^2y') - 10x - 10yy' = 3y^2y'$$

$$\Rightarrow 3x^2y^2y' - 10yy' - 3y^2y' = -2xy^3 + 10x$$

$$\Rightarrow \boxed{y' = \frac{-2xy^3 + 10x}{3x^2y^2 - 10y - 3y^2}}$$

7. (15 pts) Two sides of a triangle are 4 cm and 5 cm, respectively. The 3rd side keeps changing, as the angle between the other two sides increases at a rate of 0.5 radians per second. Find the rate at which the *area* of the triangle is changing when the angle between the sides of fixed length is $\frac{\pi}{3}$.



want $\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}}$

$\frac{h}{4} = \sin\theta$

$h = 4\sin\theta$

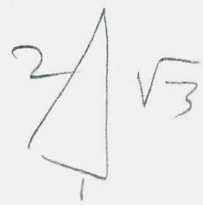
Sorry so sloppy. I scribbled it for Ken & Terry, who had passion about it.

$A = \frac{1}{2}(5)(4\sin\theta)$

$\frac{dA}{dt} = (10\cos\theta)\frac{d\theta}{dt}$

$= \left(10\cos\frac{\pi}{3}\right)(.5)$

$= 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{10}{4} = \frac{5}{2}\ \frac{cm^2}{s}$



8. (10 pts) The radius of a sphere is measured as 10 cm, with a possible error in measure of 0.15 cm. Use differentials to estimate the maximum possible error in the measurement of the volume of the sphere. Hint: The volume of a sphere is $\frac{4}{3}\pi r^3$.

$V = \frac{4}{3}\pi r^3$, Want ΔV approx.

$dV = 4\pi r^2 dr = 4\pi(10)^2(.15) = 4\pi(100)\left(\frac{15}{100}\right) \boxed{= 60\pi\ cm^3 \approx \Delta V}$

$\approx 188.49555\,92$