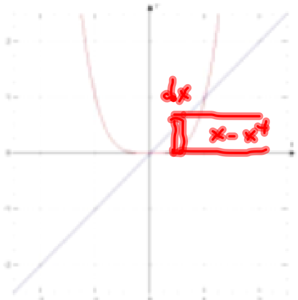
1. (20 pts.) Let R be the region bounded by the curves $y = x^4$ and $y = x$.

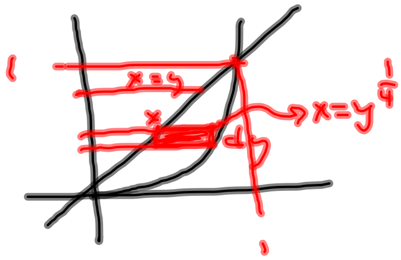
a. Set up the integral to find the area of R with respect to x. Do not evaluate



$$\int_0^1 (x - x^4)\,dx$$

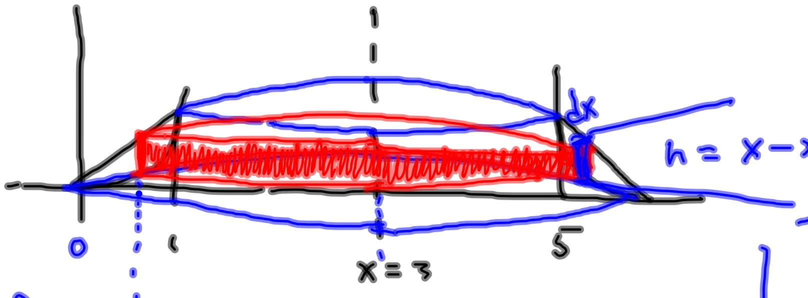
b. Set up the integral to find the area of R with respect to y. Do not evaluate

$$\int_0^1 (y^{\frac{1}{4}} - y)\,dy$$



2. (20 pts.) Set up the integral to find the volume of the solid generated when the region in problem 1. is revolved about the line x = 3.

a. Set up the integral with respect to x. Do not evaluate



$2\pi r h \Delta x$

$2\pi(3-x)(x-x^4)\Delta x$

$h = x - x^4$

Shells

$r =$

$x \longrightarrow 3$

$3-x$

$$\boxed{2\pi\int_0^1 (3-x)(x-x^4)\,dx}$$

b. Set up the integral with respect to y. Do not evaluate

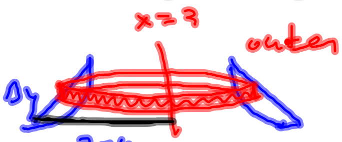
Disks (washers)

$y = x^4$

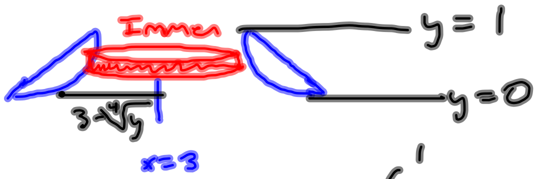
$x = y$

$x = 3-y$

$x = 3-\sqrt[4]{y}$

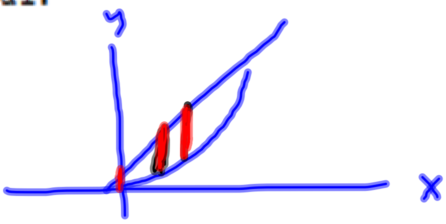


outer^2 − inner^2

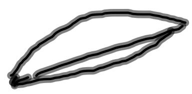


$$\pi\int_0^1 \left((3-y)^2 - \left(3-\sqrt[4]{y}\right)^2\right)dy$$

3. (10 pts.) Set up the integral to find the volume of the solid whose base is the region described in problem 1, and whose cross sections perpendicular to the x-axis are rectangles with the height equal to three times the base. Do not evaluate the integral.



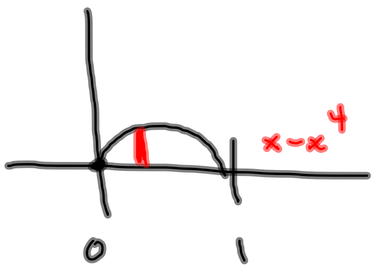
pea pod

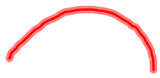


Volume of representative rectangle is:

$$3\int_0^1 (x-x^4)(x-x^4)\,dx$$

$f(x) = x-x^4$





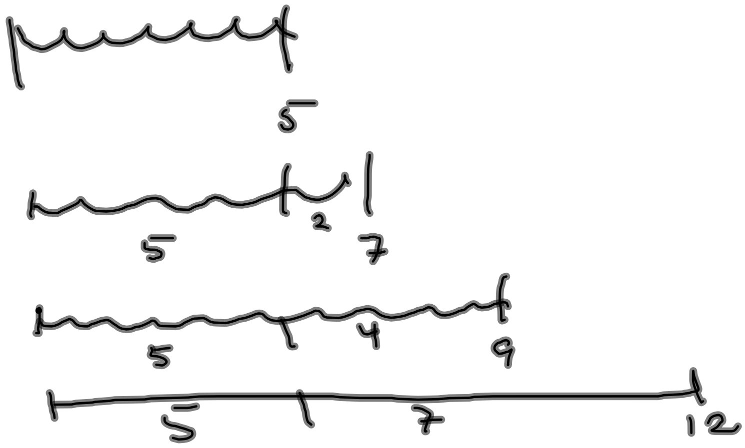
4. (15 pts.) The natural length of a spring is 5 in. If a force of 20 lb is needed to hold it at a length of 7 in., find the work done in stretching it from 9 in. to 12 in.

$F = kx$

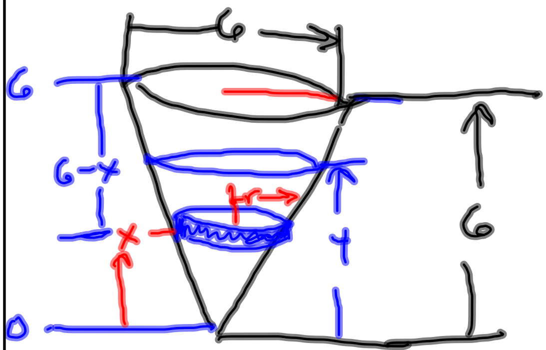
$20 = k \cdot 2$

$10 = k$

$$\int_4^7 10x\,dx$$



5. (10 pts.) A tank in the shape of a cone has a diameter across the top of 6 feet, the depth of the tank is 6 feet, and the depth of the water is 4 feet. Set up the integral to find the work required to pump all of the water out over the side. (Water weighs 62.5 lb per cubic foot.) Do not evaluate the integral.



WORK = F × Distance

$= \text{VOLUME} \times \frac{\text{FORCE}}{\text{UNIT VOL}} \times D$

$= \pi (r^2)(\Delta x) \frac{62.5 \text{ lb}}{ft^3} \times (6-x)$

r: $(0,0), (6,3)$

$(x_1, y_1), (x_2, y_2)$

$(x_1, r_1), (x_2, r_2)$

$m = \frac{1}{2}$

$r = \frac{1}{2}(x-0) + 0$

$= \frac{1}{2}x$

$$62.5\,\pi \int_0^4 \left(\tfrac{1}{2}x\right)^2 (6-x)\,dx$$

6. (10 points) Find the limit: $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 - 1}$

$$= \lim_{x \to -1} \frac{(x+2)(x+1)}{(x+1)(x-1)} = \lim_{x \to -1} \frac{x+2}{x-1} = \frac{-1+2}{-1-1} = \boxed{\frac{1}{-2}}$$

7. (15 points) Find the equation of the line tangent to the curve $y = x^3 + x^2 + 2x - 2$ at the point (1, 2)

(x_1, y_1)

$$y' = 3x^2 + 2x + 2$$

$$y'\big|_{x=1} = 3+2+2 = 7 = m_{tan}$$

$$y = m(x - x_1) + y_1$$

$$y = 7(x-1) + 2$$

8. (10 points) Calculate y' if $y = \tan(x^5 - 2x + 3\sqrt{x})$.

$$= \tan(x^5 - 2x + 3x^{\frac{1}{2}}) \Longrightarrow$$

$$y' = \sec^2(x^5 - 2x + 3x^{\frac{1}{2}}) \cdot (5x^4 - 2 + \tfrac{3}{2}x^{-\frac{1}{2}})$$

9. (15 points) Calculate y' if $x \cos y = y \sin x$

$$(-\sin y)y' = y'\sin x + y\cos x$$

$$y'(-\sin y - \sin x) = y\cos x$$

$$y = \frac{y\cos x}{-\sin y - \sin x}$$

No, Dr. Steve. You blew the product rule on the left-hand side of the equation.
Maybe you should've used that '1' that Beth wants you to use, man:
1 cos(y) + x (- sin(y)) y' = y' sin(x) + y cos(x)
The above is what the first step should look like, rather than the junk I did.

10. (10 points) Find the limit: $\lim_{x\to\infty}(\sqrt{x^2+3x}-x)$

$$\left(\sqrt{x^2+3x}-x\right)\left(\frac{\sqrt{x^2+3x}+x}{\sqrt{x^2+3x}+x}\right)=\frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x}$$

$$=\frac{3x}{\sqrt{x^2+3x}+x}=\frac{x(3)}{x\left(\sqrt{1+\frac{3}{x}}+1\right)}=\frac{3}{\sqrt{1+\frac{3}{x}}+1}$$

$$\xrightarrow{x\to\infty}\frac{3}{\sqrt{1}+1}=\frac{3}{2}$$

11. (10 points) If $f'(t) = 3t^2 - \cos t$, and $f(0)= 4$, find $f(t)$

$f(t)=t^3-\sin t+c$

$f(0)=0^3-\sin(0)+c=4 \longrightarrow c=4$

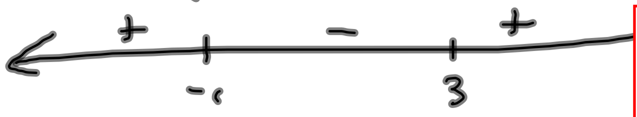
$\boxed{f(t)=t^3-\sin t+4}$

12. (20 points) Given $f(x) = 2x^3 - 6x^2 - 18x - 2$

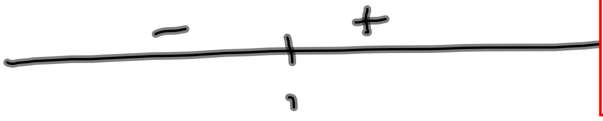
a. Make a chart showing where the first and second derivatives are positive and where they are negative.

$f(x) = 2(x^3 - 3x^2 - 9x - 1)$

$f'(x) = 2(3x^2 - 6x - 9) = 6(x^2 - 2x - 3) = 6(x-3)(x+1)$

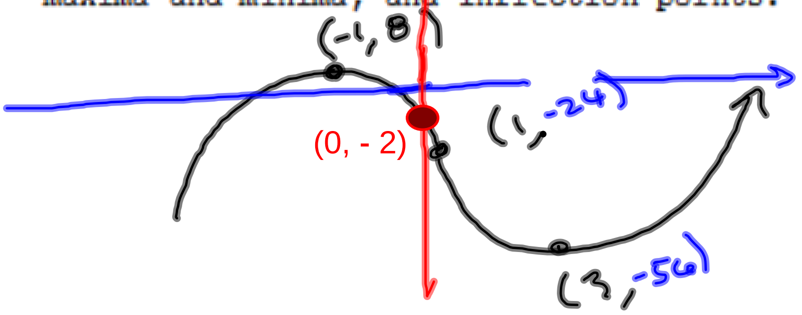


$f''(x) = 6(2x - 2) = 12(x-1)$



Took me close to 5 minutes to realize I didn't necessarily have to find the *x*-intercepts for *f* (*x*); rather, we just needed to find the critical values and sign pattern for *f* ' and *f* '' and guesstimate the *x*-intercepts with a little help from the *y*-intercept and the "shape" of *f*.

b. Sketch the graph (you may have to use different scales for the x axis and y axis.) Give the coordinates of all relative maxima and minima, and inflection points.



$$\begin{array}{r|rrrr} -1 & 2 & -6 & -18 & -2 \\ & & -2 & 8 & 10 \\ \hline & 2 & -8 & -10 & 8 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -6 & -18 & -2 \\ & & 2 & -4 & -22 \\ \hline & 2 & -4 & -22 & -24 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 2 & -6 & -18 & -2 \\ & & 6 & 0 & -54 \\ \hline & 2 & 0 & -18 & -56 \end{array}$$

13. (10 points) Evaluate $\int_0^5 \sqrt{25-x^2}\,dx$ by interpreting it as an area.

$$\boxed{\frac{1}{2}\pi(5)^2}$$

Yet another blunder, under time crunch. This is the RIGHT half of the TOP half of a circle of radius 5, centered at the origin. The above is off by a factor of 2.

14. (15 points) Evaluate the integral $\frac{1}{4}\int_{-1}^{2} 4x^3\sqrt{x^4+3}\,dx$

$u = x^4+3$

$du = 4x^3\,dx$

$$= \frac{1}{4}\left[\frac{2}{3}(x^4+3)^{\frac{3}{2}}\right]_{-1}^{2}$$

$$= \boxed{\frac{1}{6}\left[19^{\frac{3}{2}} - 8\right]}$$

15. (10 points) Find the derivative of f(x) = $\int_1^{x^2} \cos(t^3)dt$

$f'(x) = \cos\left((x^2)^3\right)(2x)$