$$\int \cos(t^2)\,dt$$

$u=t^2 \Rightarrow du = 2t\,dt$, which we don't have. And we can't make it, because there's no factor of t to play with.

$$\int t\cos(t^2)\,dt = \frac{1}{2}\int \cos(t^2)(2t\,dt)$$

$u=t^2$, $du=2t\,dt$

$$\int (2x^3-3x)\sqrt[3]{x^4-3x^2}\,dx$$

($u = x^4-3x^2$ $\quad du=(4x^3-6x)\,dx$

we have $2x^3-3x$ to play with.)

Hmmmmmm $2(2x^3-3x) = 4x^3-6x$ sweet!

$$=\frac{1}{2}\int (x^4-3x^2)^{\frac{1}{3}}\underbrace{(2(2x^3-3x))\,dx}_{du}$$

$$\int_0^{\frac{T}{2}} \sin\left(\frac{2\pi t}{T-\theta}\right) dt$$

t is the variable with respect to which we are integrating.

$$= \int_0^{\frac{T}{2}} \sin\left(\left(\frac{2\pi}{T-\theta}\right)t\right) dt$$

t is the variable we're integrating with respect to.

$$\left(u = \frac{2\pi}{T-\theta} t \Rightarrow du = \frac{2\pi}{T-\theta} dt\right)$$

$$\left(t=0 \Rightarrow u=0,\ t=\frac{T}{2} \Rightarrow u = \frac{2\pi}{T-\theta}\cdot\frac{T}{2} = \frac{\pi T}{T-\theta}\right)$$

$$= \frac{T-\theta}{2\pi}\int_0^{\frac{T}{2}} \sin\left(\frac{2\pi}{T-\theta}t\right)\left(\frac{2\pi}{T-\theta}dt\right) = \frac{T-\theta}{2\pi}\int_0^{\frac{\pi T}{T-\theta}} \sin(u)\, du$$

$$= \frac{T-\theta}{2\pi}\Big[-\cos(u)\Big]_0^{\frac{\pi T}{T-\theta}} = \frac{T-\theta}{2\pi}\left[-\cos\left(\frac{\pi T}{T-\theta}\right)+\cos(0)\right]$$

$$= \frac{T-\theta}{2\pi}\left[1-\cos\left(\frac{\pi T}{T-\theta}\right)\right]$$

Apr 27-12:22 PM

$$\int_0^{\frac{T}{2}} \sin\left(\frac{2\pi t}{T-\theta}\right) dt$$

t is the variable with respect to which we are integrating.

$$= \int_0^{\frac{T}{2}} \sin\left(\left(\frac{2\pi}{T-\theta}\right)t\right) dt$$

t is the variable we're integrating with respect to.

$$\left(u = \frac{2\pi}{T-\theta}t \implies du = \frac{2\pi}{T-\theta}\,dt\right)$$

$$\cancel{\left(t=0 \implies u=0,\ t=\frac{T}{2} \implies u=\frac{2\pi}{T-\theta}\cdot\frac{T}{2}=\frac{\pi T}{T-\theta}\right)}$$

This gives us $\frac{T-\theta}{2\pi}\int \sin\left(\frac{2\pi}{T-\theta}t\right)\left(\frac{2\pi}{T-\theta}dt\right)$

$$= \frac{T-\theta}{2\pi}\left[-\cos\left(\frac{2\pi}{T-\theta}t\right)\right] + C$$

So, the definite integral is $\frac{T-\theta}{2\pi}\left[-\cos\left(\frac{2\pi}{T-\theta}t\right)\right]_0^{\frac{T}{2}} =$

This entailed no change to upper & lower limits of integration.

$$\frac{T-\theta}{2\pi}\left[-\cos\left(\frac{2\pi}{T-\theta}\cdot\frac{T}{2}\right) - \left(-\cos\left(\frac{2\pi}{T-\theta}\cdot 0\right)\right)\right]$$

$$= \frac{T-\theta}{2\pi}\left[1 - \cos\left(\frac{\pi T}{T-\theta}\right)\right]$$

$$\int x^3\sqrt{x^2+1}\,dx$$

$u = x^2+1 \implies du = 2x\,dx$

$$= \frac{1}{2}\int x^2\sqrt{x^2+1}\cdot\underbrace{2x\,dx}_{du} =$$

$u = x^2+1 \implies$

$x^2 = u-1$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int (u-1)\,u^{\frac{1}{2}}\,du$$

$$\int \frac{x}{(x^2+1)^2}\,dx \qquad u = x^2+1, \quad du = 2x\,dx$$

$$= \int (x^2+1)^{-2}\, x\, dx = \frac{1}{2}\int (x^2+1)^{-2}(2x\,dx)$$

$$= \frac{1}{2}\int u^{-2}\,du$$

$$\int \sec^3 x \tan x\, dx = \int \sec^2 x \sec x \tan x\, dx$$

$$u = \sec x \Longrightarrow du = \sec x \tan x\, dx$$

$$= \int u^2\, du = \frac{1}{3}u^3 + c = \frac{1}{3}\sec^3 x + C$$

$$\int \sec^3 x \tan x\, dx = \int \frac{1}{\cos^3 x}\cdot\frac{\sin x}{\cos x}\,dx$$

$$= \int \frac{1}{\cos^4 x}\cdot \sin x\, dx = \int \cos^{-4} x \cdot \sin x\, dx$$

$$u = \cos x \Longrightarrow du = -\sin x\, dx$$

$$= -\int (\cos^{-4} x)(-\sin x\, dx)$$

$$= -\int u^{-4}\,du = -\frac{1}{-3}u^{-3} + c = \frac{1}{3}\cos^{-3} x + C$$

$$= \frac{1}{3}\sec^3 x + C$$

$$\cancel{\int \frac{1}{x}\,dx = \ln|x| + C}$$

Prove that $\lim_{x\to 3}(5x-7)=8$

Scratch:

$\boxed{|5x-7-8|<\varepsilon}$

$|5x-15|<\varepsilon$

$5\boxed{|x-3|}<\varepsilon$

$|x-3|<\frac{\varepsilon}{5}\equiv\delta$

Proof

Let $\varepsilon>0$ be given. Define $\delta=\frac{\varepsilon}{5}$. Then for all x such that $0<|x-3|<\delta$, we have

$|5x-7-8|=|5x-15|$

$=5|x-3|<5\delta=5\cdot\frac{\varepsilon}{5}=\varepsilon$

∎

Apr 27-12:51 PM

$$\int \frac{\cos(x^{-2})}{x^3}\,dx$$

$$u = x^{-2} \implies du = -2x^{-3}\,dx$$

$$-\tfrac{1}{2}x^{3}\,du = dx$$

$$= -\tfrac{1}{2}\int \cos(x^{-2})\,(-2x^{-3}\,dx)$$

$$= -\tfrac{1}{2}\int \cos(u)\,du = -\tfrac{1}{2}\sin(u) + C$$

$$= -\tfrac{1}{2}\sin(x^{-2}) + C$$

Apr 27-12:58 PM