$$\text{Work} \approx \sum_{k=1}^{n} f(x_k)\Delta x \approx \int_a^b f(x)\,dx = \text{Work},$$

where $f(x)$ is the force applied, as a function of position x. Add up all those forces-times-small-distances in the sum $\left(\sum\right)$ and you get an estimate on the total amount of work. Assuming the force is described by a continuous function, you can find the *precise* amount of work with the integral $\left(\int\right)$. If force is given in Newtons, and position is measured in meters, then the result is in Newton*meters (traditionally written as Newton-meters).

In the context of work, though, we replace the term Newton-meter with Joules (force in the same direction as the motion), and when we talk about Newton-meters (or foot-pounds), it's usually reserved for torque, and the force is measured at right angles to the distance measure. Strictly speaking, both are given units of

$$\frac{kg \cdot m^2}{s^2} = kg \cdot \frac{m}{s^2} \cdot m = \text{mass*acceleration*distance}$$

When you're turning a screw with a wrench, you use Newton-meters, where you're applying the force at right angles to the handle of the wrench. Torque measures the tendency of the screw to turn – How hard you're pushing times the length of the wrench measures the torque. You'll learn more about torque (much more than this oversimplification) in Calculus III.

The RATE at which work is being done is Joules per second, $\left(\frac{kg \cdot m^2}{s^3}\right)$, also known as Watts.

§6.4 #16

bucket weighs 4 lbs

well is 80 ft deep

40 lbs of water

Pulled up at 2 ft/s

water leaking @ $\frac{.2\text{ lb}}{s}$

Find work done

$F \cdot D = \text{work}.$

$$\frac{80\text{ft}}{\frac{2\text{ft}}{s}} = 40s$$

Force = 4 lbs + 40 lbs = $F(0)$

$$F(t) = 44 - \frac{.2\text{lb}}{s} \cdot t$$

$$= \boxed{44 - .2t}$$

Distance = 2t

$$\int_0^{40} (44 - .2t)\underbrace{(2t\,dt)}_{?}$$

$F = 44 - .2t$

$D = 2t$

Bucket: $(4\,\text{lb})(80\,\text{ft}) = 320$ ft-lbs

x_k $F(x_k) = 40 - .2t = 40 - .2\left(\frac{1}{2}x_k\right)$

$$\boxed{x_k = 2t \Longrightarrow t = \frac{1}{2}x_k}$$

Work done over distance Δx is

$(40 - .1x_k)\Delta x$ ft-lbs of work.

To lift the water 80 ft

$$W \approx \sum (40 - .1x)\Delta x \approx \int_0^{80} (40 - .1x)\,dx$$

$$= \left[40x - .1\frac{x^2}{2}\right]_0^{80} = 40(80) - .05(80)^2$$

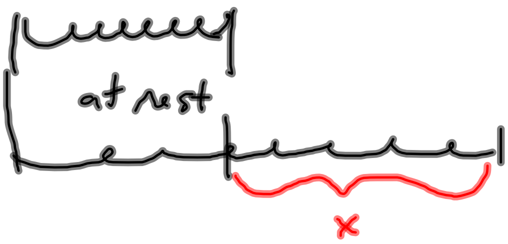
$$= 3200 - .05(6400) = 3200 - \frac{5}{100}(6400)$$

$$= 3200 - 320 = 2880 \text{ ft-lbs.}$$ For the water alone.

Add 320 ft-lbs for the work lifting the bucket, which gives 3200 ft-lbs.

Hooke's Law: Force required to keep a spring stretched is proportional to the amount its been stretched.

$$F = Kx$$



#10 work required to stretch a spring 1 ft is 12 ft-lbs. How much work to stretch it 9 inches $= \frac{3}{4}$ ft beyond its natural length.

$$F = kx$$

$$\int_0^1 F\,dx = 12$$

Find $\int_0^{.75} F\,dx$

$$\int_0^1 kx\,dx = 12$$

$$\frac{k}{2}x^2\Big]_0^1 = 12$$

$$\frac{k}{2} = 12$$

$$k = 24$$

$$\int_0^{.75} 24x\,dx = 12x^2\Big]_0^{.75} = 12\left(\frac{9}{16}\right) = \frac{3(9)}{4}$$

$$= \frac{27}{4} = 6.75 \text{ ft-lbs}$$

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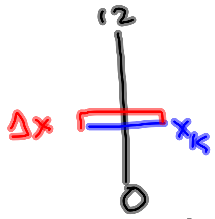
(17) leaky 10-kg bucket



Speed = constant

Rope weighs $\frac{.8\,kg}{m}$

Contains 36 kg @ start

Emptied RIGHT when bucket reaches 12 m.

$$(46 - 3x_k)\left(9.8\,\frac{m}{s^2}\right)\Delta x$$
$$+\,(12(.8) - .8x_k)\left(9.8\,\frac{m}{s^2}\right)\Delta x$$

$(t_1, m_1) = (0, 36)\ ,\ (12, 0) = (t_2, m_2)$

$1\,\frac{kg\text{-}m}{s^2} = 1\,N$

$\frac{36}{12}$ = rate of leakage in $\frac{kg}{m}$

$(12)\left(\frac{8}{10}\right) = \frac{6 \cdot 8}{5} = \frac{48}{5}$

$$\underbrace{(46)\left(\frac{9.8\,m}{s^2}\right)}_{F} - 3\,\frac{kg}{m}\cdot\frac{9.8\,m}{s^2}$$

$$9.8\int_0^{12}\big((46-3x) + (9.6 - .8x)\big)\,dx = 9.8\int_0^{12}(55.6 - 3.8x)\,dx$$

$$= 3857\ \text{"}N\text{-}m\text{"}$$

→ Joules