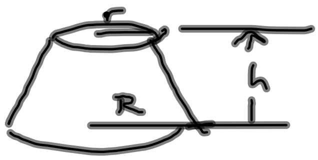
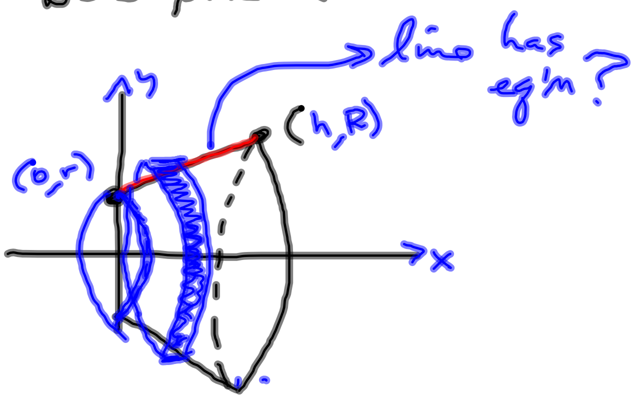
§6.2 #50

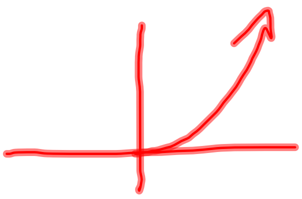
I advise MORE §5.5 practice

Find Vol of





&. y & x ≥ 0



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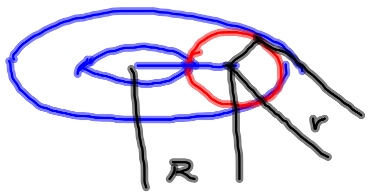
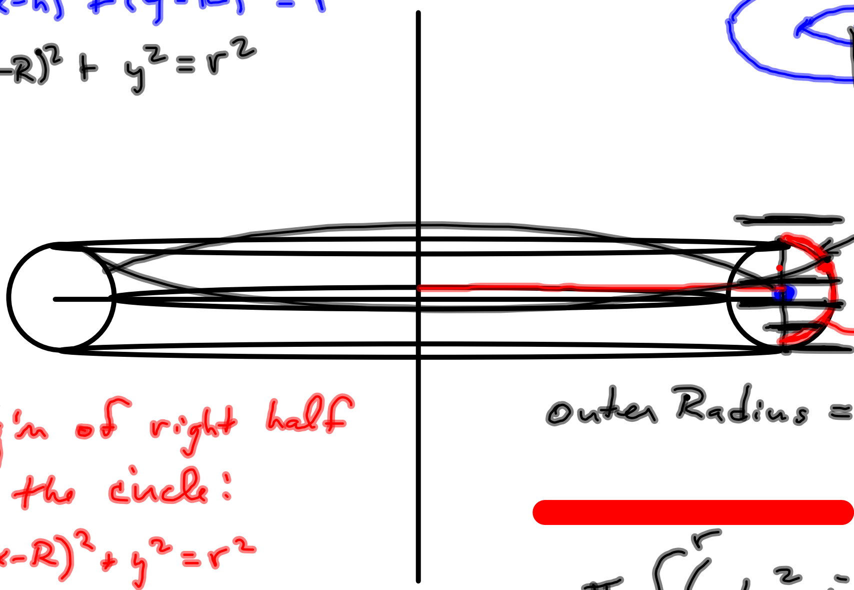
6.2 #63

$(x-h)^2+(y-k)^2=r^2$

$(x-R)^2+y^2=r^2$





has equation

$\to R+\sqrt{r^2-y^2}$

Eq'n of right half of the circle:

$(x-R)^2+y^2=r^2$

$(x-R)^2=r^2-y^2$

$x-R=\pm\sqrt{r^2-y^2}$

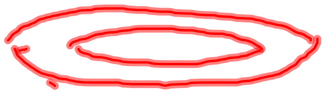
$x=R\pm\sqrt{r^2-y^2}$

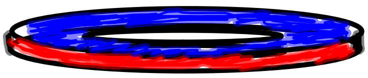
$R+\sqrt{r^2-y^2}$ is Right half

$R-\sqrt{r^2-y^2}$ is Left half

Outer Radius =

$$\pi\int_{-r}^{r}(\text{outer}^2-\text{inner}^2)\,dy$$





Volume of washer is volume of outer disk − " " − inner disk

$$\pi r_1^2 h-\pi r_2^2 h$$

$$=\pi r_1^2\Delta y-\pi r_2^2\Delta y$$

$$=\pi(r_1^2-r_2^2)\Delta y$$

$$=\pi\left(\left(R+\sqrt{r^2-y^2}\right)^2-\left(R-\sqrt{r^2-y^2}\right)^2\right)\Delta y$$

$$\text{Volume}=\pi\int_{-r}^{r}\left(\left(R+\sqrt{r^2-y^2}\right)^2-\left(R-\sqrt{r^2-y^2}\right)^2\right)dy$$

$$\text{Volume} = \pi\int_{-r}^{r}\left(\left(R+\sqrt{r^2-y^2}\right)^2 - \left(R-\sqrt{r^2-y^2}\right)^2\right)dy$$

$$= \pi\int_{-r}^{r}\left[R^2 + 2R\sqrt{r^2-y^2} + (r^2-y^2) - \left(R^2 - 2R\sqrt{r^2-y^2} + (r^2-y^2)\right)\right]dy$$

$$= \pi\int_{-r}^{r} 4R\sqrt{r^2-y^2}\,dy = 4\pi R\int_{-r}^{r}\sqrt{r^2-y^2}\,dy$$

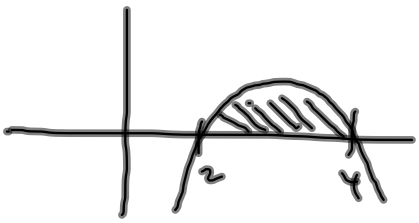
<u>Daniel</u>

$$= 4\pi R\left[\tfrac{1}{2}\pi r^2\right] = 2\pi^2 r^2 R$$

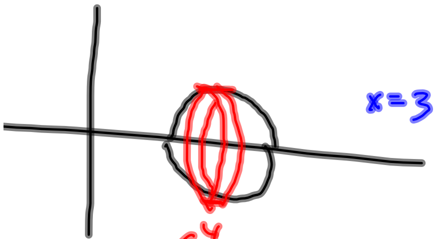
§6.3 #38

$y = -x^2 + 6x - 8$, $y = 0$ about the x-axis

$y = -(x^2 - 6x + 8)$



$x = 3$



$\pi \int_2^4$

$(a-b)^2 = a^2 - 2ab + b^2$

$(-x^2+6x-8)^2 = (x^2-6x+8)^2$

$= ((x^2-6x)+8)^2$

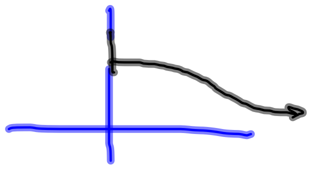
$= (x^2 - (6x-8))^2$

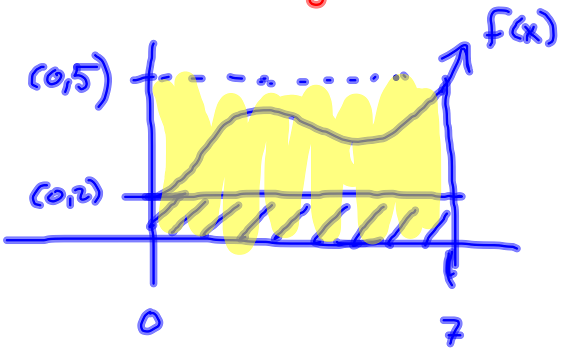
$= x^4 - 2x^2(6x-8) + (6x-8)^2$

$= x^4 - 12x^3 + 16x^2 + 36x^2 - 96x + 64$

$= x^4 - 12x^3 + 52x^2 - 96x + 64$

$$\frac{2}{5} \le \int_0^2 \frac{1}{x^2+1}\,dx \le 2$$





$$14 \le \int_0^7 f(x)\,dx \le 35$$

$$m = \min\{f(x) \mid x \in [0,7]\}$$
$$= \min_{[0,7]}\{f(x)\}$$

$$M = \max_{[0,7]}\{f(x)\} = 5$$

Max/Min Principle:

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Claim:

$$\frac{2}{5} \le \int_0^2 \frac{1}{x^2+1}\,dx \le 2 \qquad \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f(x) = \frac{1}{x^2+1} = (x^2+1)^{-1} \Longrightarrow f'(x) = -(x^2+1)^{-2}(2x)$$

$$= -\frac{2x}{(x^2+1)^2} < 0 \text{ when } x>0$$

$f(x)$ is decreasing on $[0,2]$

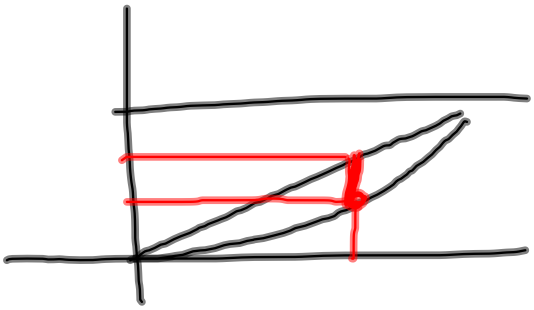
$$M = f(0) = 1$$

$$m = f(2) = \frac{1}{2^2+1} = \frac{1}{5}$$

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

$$\frac{1}{5}(2) \le \int_0^2 \frac{1}{x^2+1}\,dx \le 1\cdot 2$$

$$\frac{2}{5} \le \int_0^2 \frac{1}{x^2+1}\,dx \le 2$$



fallacious