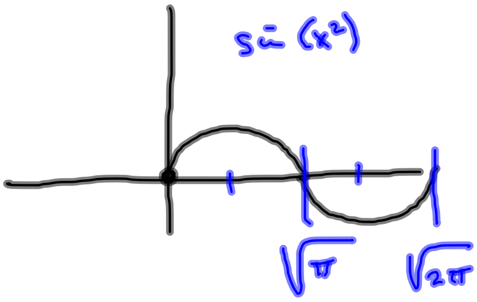
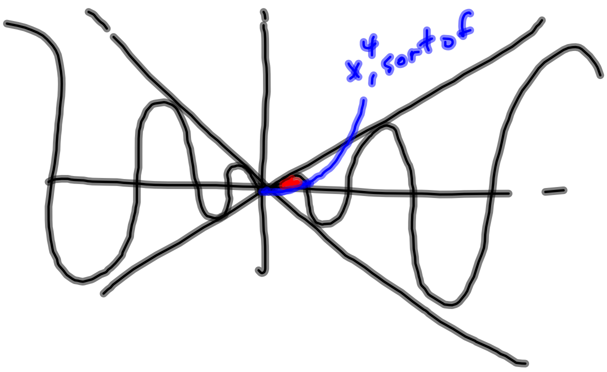
§6.1 # 35 sort of

$y = x\sin(x^2)$, $y = x^4$





$x\sin(x^2) = x^4$

$x(\sin(x^2) - x^3) = 0$

$x = 0$

$\sin(x^2) - x^3 = 0$

Graphing Calculator is ~~almost~~ essential for this one.

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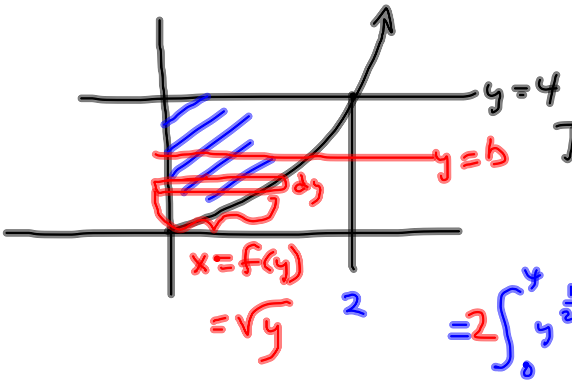
$y = x^2$, $y = 4$

If you can do this for $0 \le x \le 2$, it's easier. And symmetry allows that.

$y = x^2$

$\pm\sqrt{y} = x$



$$\text{Total Area} = \int_{y=0}^{y=4} \sqrt{y}\, dy$$

$$= 2\int_0^4 y^{\frac{1}{2}} dy = 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = 2\left[\frac{2}{3} y^{\frac{3}{2}}\right]_0^4$$

$$= 2 \cdot \frac{2}{3}\left[4^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = 2 \cdot \frac{2}{3}\left[2^3\right] = 2 \cdot \frac{2}{3}[8] = \frac{16}{3} \cdot 2$$

we want $\frac{1}{2}$ of that: $\frac{8}{3} \cdot 2$

The question becomes

Solve $\int_0^b \sqrt{y}\, dy = \frac{8}{3} \cdot 2$ for b

6.1 Due Wed.

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ Binomial Theorem

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\binom{5}{3} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{(2\cdot 1)\cancel{(3\cdot 2\cdot 1)}}$$

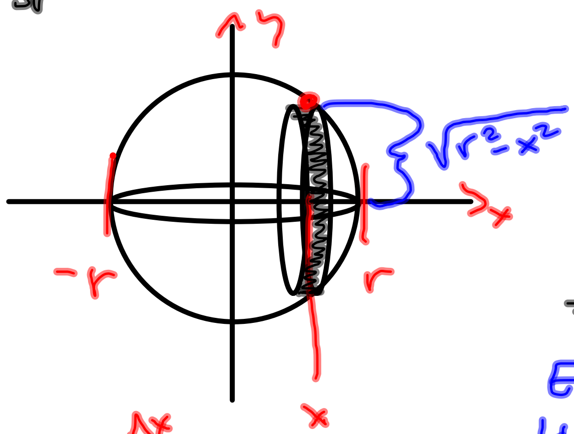
$$= \frac{20}{2} = 10$$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}$$

$$(x-2)^5 = 1(x)^5(-2)^0 + 5(x)^4(-2)^1 + 10(x)^3(-2)^2$$
$$+ 10(x)^2(-2)^3 + 5(x)^1(-2)^4 + 1(x)^0(-2)^5$$

$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

Sphere of radius r.



Let $A(x)$ = cross-sectional area of a disc.

Let Δx be its thickness

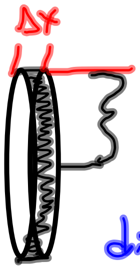
Then the volume of this cylinder is

Equation of the circle in the xy-plane

$$x^2+y^2=r^2$$

$$\vdots$$

$y=\sqrt{r^2-x^2}$ is its top half.



Volume of the disc is $\pi\left(\sqrt{r^2-x^2}\right)^2\Delta x$

$$= \pi(r^2-x^2)\Delta x$$

Volume of the sphere

$$= V \approx \sum_{k=1}^{n} \pi(r^2-x_k^2)\Delta x \xrightarrow{\Delta x \to 0} \int_{-r}^{r} \pi(r^2-x^2)\,dx$$

$$= \pi\left[r^2x - \frac{x^3}{3}\right]_{-r}^{r}$$

$$= \pi\left[r^2\cdot r - \frac{r^3}{3} - \left(r^2\cdot(-r) - \frac{(-r)^3}{3}\right)\right]$$

$$= \pi\left[r^3 - \frac{r^3}{3} + r^3 - \frac{r^3}{3}\right]$$

$$= \pi\left[2r^3 - \frac{2r^3}{3}\right] = \pi\left[\frac{6r^3-2r^3}{3}\right] = \pi\left[\frac{4r^3}{3}\right]$$

$$= \frac{4\pi r^3}{3}$$

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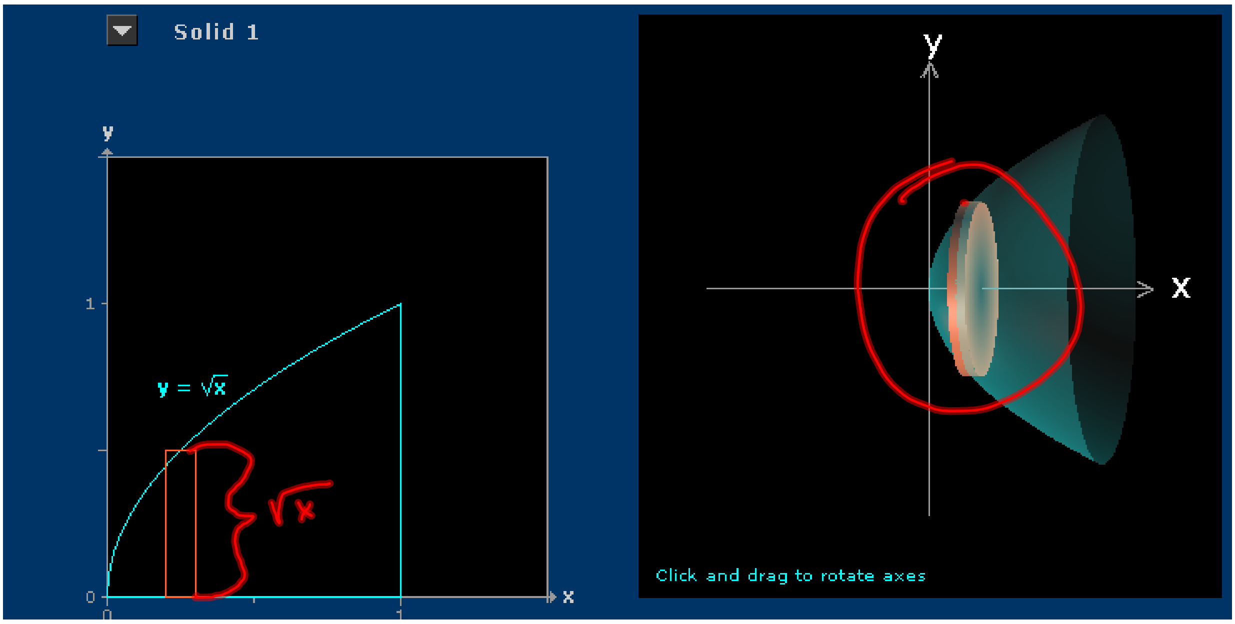

$$V = \int_0^1 \pi(\sqrt{x})^2\,dx = \pi\int_0^1 x\,dx$$

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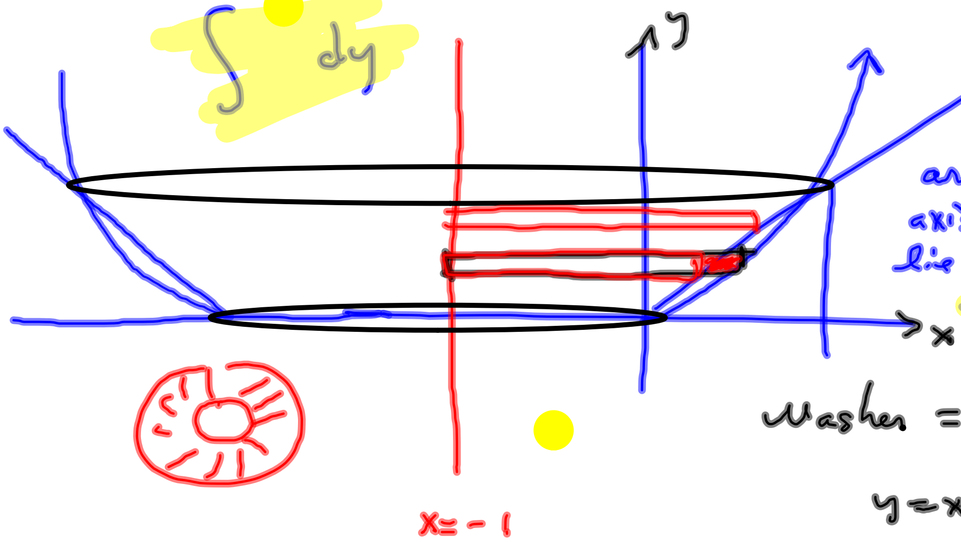
Revolve the region bdd by, $x=0$, $x=1$
$y=x^2$, $y=x$ around the line $x=-1$



we're revolving around a vertical axis. Our washers lie flat and we integrate with respect to y.

Washer = outer − inner

$y=x^2 \Longrightarrow x=\sqrt{y}$

Vertical March dy

outer $= \sqrt{y} - (-1)$

inner $= y - (-1)$

Volume : $\left(\pi(\sqrt{y}+1)^2 - \pi(y+1)^2\right)\Delta y$

$$V = \pi \int_{0=y}^{1=y} \left((\sqrt{y}+1)^2 - (y+1)^2\right) dy$$