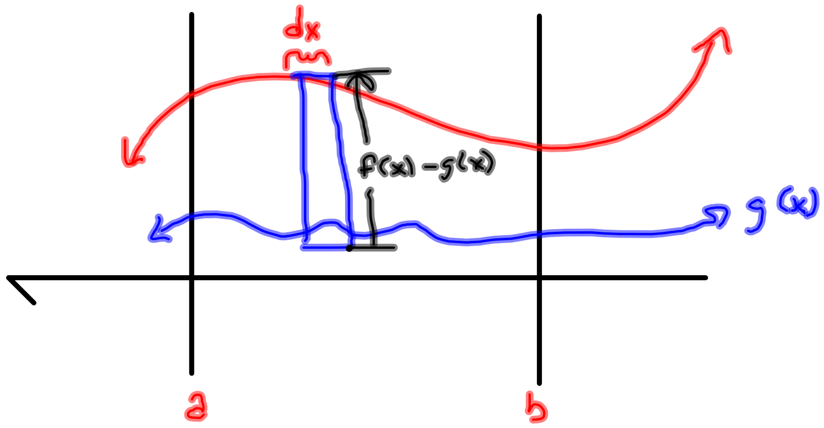
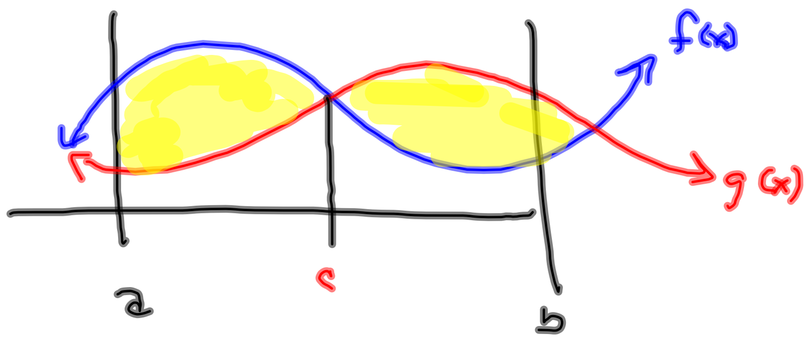
Area between f(x) and g(x) if $f(x) \geq g(x)$ on [a,b] is

$$\int_a^b [f(x) - g(x)]\,dx$$



In general, area = $\int_a^b |f(x) - g(x)|\,dx$



$$\text{Area} = \int_a^b |f(x) - g(x)|\,dx = \int_a^c (f(x) - g(x))\,dx + \int_c^b (g(x) - f(x))\,dx$$

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Key skills

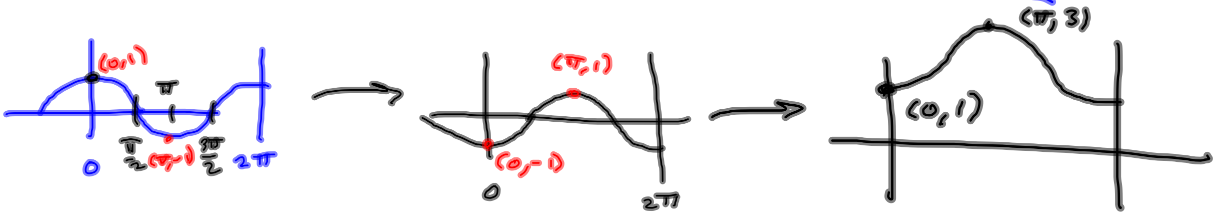
(1) Graphing

(2) Equation solving

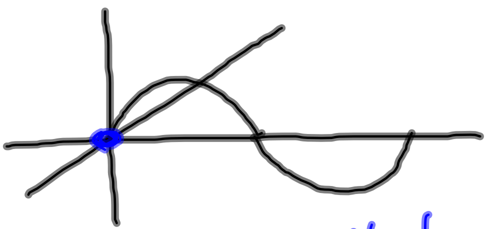
we have technology backup. But do what you can, by hand.

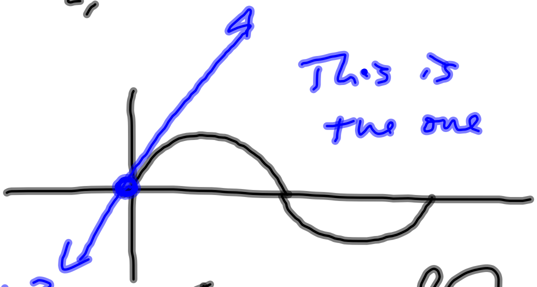
$$2-\cos(x) = -\cos x + 2$$

$$\cos x \longrightarrow -\cos x \longrightarrow -\cos x + 2$$



(6) $y = \sin x$, $y = x$, $x = \frac{\pi}{2}$, $x = \pi$





This is the one

Which picture?

$f(x) = \sin x$

$f'(x) = \cos x$

$f'(0) = \cos(0) = 1$

$g(x) = x$

$g'(x) = 1$

Same slope @ $x = 0$.

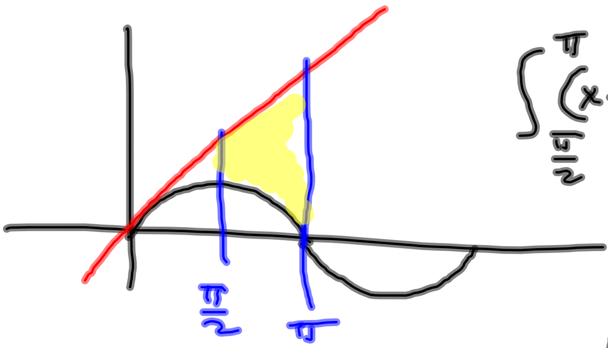
Same y-val @ $x=0$

Same slope @ $x=0$

$\sin x$ is <u>concave</u> <u>down</u> and $y=x$ is increasing at a constant rate, so it's <u>Above</u> $\sin x$ for $x > 0$.

$f''(x) = -\sin x < 0$ on $(0, \pi)$

⑥ $y = \sin x$, $y = x$, $x = \frac{\pi}{2}$, $x = \pi$



$$\int_{\frac{\pi}{2}}^{\pi} (x - \sin x)\,dx$$

$$= \left[\frac{1}{2}x^2 + \cos x\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{1}{2}\pi^2 + \cos(\pi) - \left[\frac{1}{2}\left(\frac{\pi}{2}\right)^2 + \cos\left(\frac{\pi}{2}\right)\right]$$

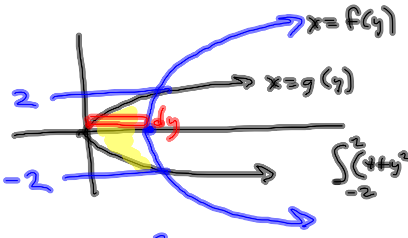
$$= \frac{\pi^2}{2} - 1 - \left[\frac{\pi^2}{8} + 0\right]$$

$$= \frac{\pi^2}{2} - 1 - \frac{\pi^2}{8} = \frac{4\pi^2 - \pi^2 - 8}{8} = \boxed{\frac{3\pi^2}{8} - 1}$$

$x = 2y^2$, $x = 4 + y^2$

To learn to graph these:

(1) Swap variables, do it sideways.

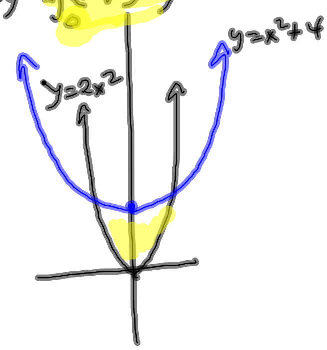


$$\int_{-2}^{2}(4+y^2-2y^2)\,dy = 2\int_{0}^{2}(4-y^2)\,dy$$

$2y^2 = 4+y^2$

$y^2-4=0$

$y=\pm 2$



(2) Solve for y & graph two functions of x:

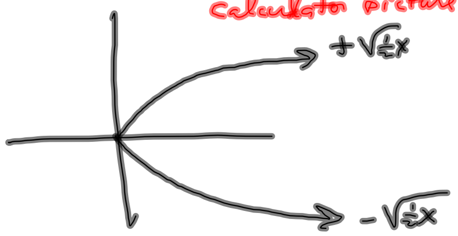
$x = 2y^2$

$2y^2 = x$

$y^2 = \frac{1}{2}x$

$y = \pm\sqrt{\frac{1}{2}x}$

This is good for graphing calculator pictures.

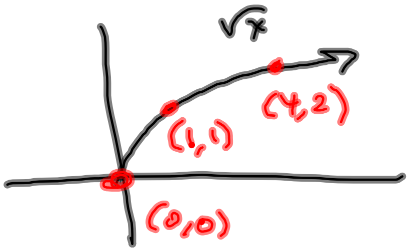


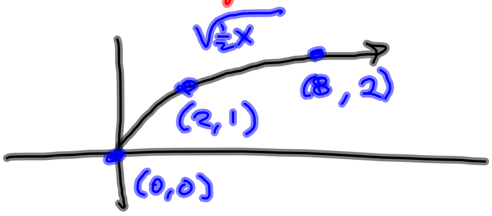
$(x,y) \longrightarrow (2x,y)$

$f(x)=\sqrt{x} \longrightarrow \sqrt{\tfrac{1}{2}x} = f(\tfrac{1}{2}x)$

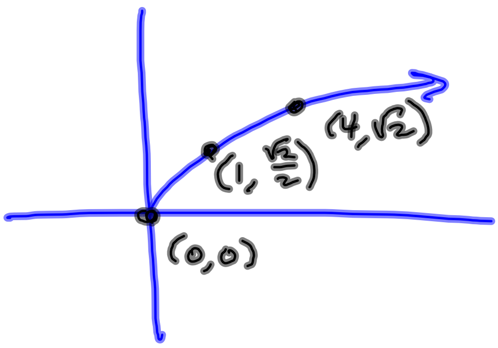
$= \sqrt{\tfrac{1}{2}}\sqrt{x} = \sqrt{\tfrac{1}{2}}f(x)$

$(x,y) \longrightarrow (x, \sqrt{\tfrac{1}{2}}\,y)$





$\sqrt{\tfrac{1}{2}}\ \sqrt{x}$



$2\sqrt{\tfrac{1}{2}} = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$\sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\cancel{\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \text{etc.}}$

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$$\frac{d}{dx}\int_0^{x^2}\sqrt{1+r^3}\,dr \quad = \quad \frac{d}{dx}\int_0^{x^2}\sqrt{1+r^3}\,dr = g(u(x))$$

$x^2 \rightarrow u(x)$

$$h(x) = \int_0^{x}\sqrt{1+r^3}\,dr \qquad = \underbrace{\sqrt{1+(x^2)^3}}_{g'(u)} \cdot \underbrace{2x}_{u'(x)}$$

$$h(u(x)) = \int_0^{u(x)}\sqrt{1+r^3}\,dr \qquad \frac{dg}{du}\cdot\frac{du}{dx}$$

See Catherine's #14, §5.3.

$$h'(x) = \frac{d}{du}\int_0^{u}\sqrt{1+r^3}\,dr \cdot \frac{du}{dx} = \sqrt{1+u^3}\cdot 2x$$

$$= \sqrt{1+(x^2)^3}\cdot 2x$$