

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n(2n^2+3n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$= \boxed{\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}} = \frac{n^3 + \text{less degree}}{3}$$

$$\int_0^2 x^2 dx \quad \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

$$x_k = a + k\Delta x = 0 + k \cdot \frac{2}{n} = \frac{2}{n}k$$

$$\text{Area} \approx \sum_{k=1}^n f(x_k) \Delta x = \sum_{k=1}^n x_k^2 \Delta x = \sum_{k=1}^n \left(\frac{2}{n}k\right)^2 \cdot \frac{2}{n} = \frac{2}{n} \sum_{k=1}^n \frac{4k^2}{n^2}$$

$$= \frac{8}{n^3} \sum_{k=1}^n k^2 = \frac{8}{n^3} \left[\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right]$$

$$= \frac{8n^3}{3n^3} + \frac{8n^2}{2n^3} + \frac{8n}{6n^3} \xrightarrow{n \rightarrow \infty} \frac{8}{3} + 0 + 0$$

$\rightarrow 0$ as $n \rightarrow \infty$

$$\int_a^b f(x) dx$$

$$25^{\frac{3}{2}} = 5^3 = 125$$

$$\int_1^x f(t) dt = \int_1^x t dt = - \int_x^1 t dt$$

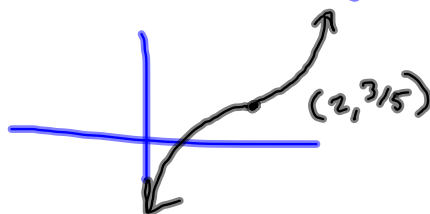
$$f(x) = (x-2)^{\frac{2}{3}}$$

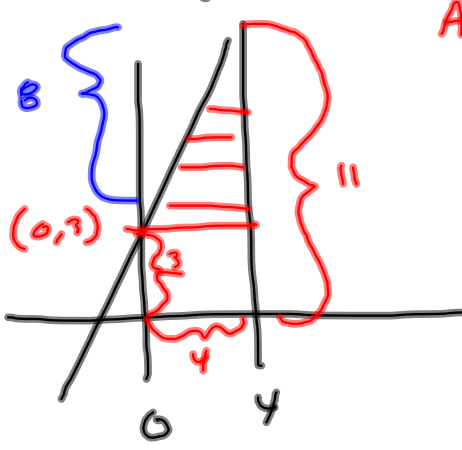
$$\int_1^x (x-2)^{\frac{2}{3}} dx = \left[\frac{3}{5} (x-2)^{\frac{5}{3}} \right]_1^x$$



$$= \frac{3}{5} (x-2)^{\frac{5}{3}} - \frac{3}{5} (-1)^{\frac{5}{3}}$$

$$= \frac{3}{5} (x-2)^{\frac{5}{3}} + \frac{3}{5}$$



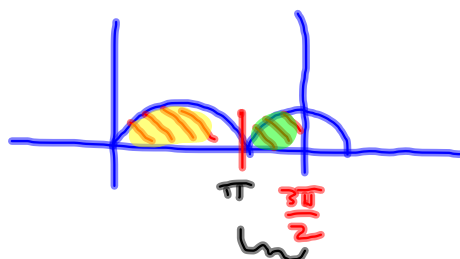
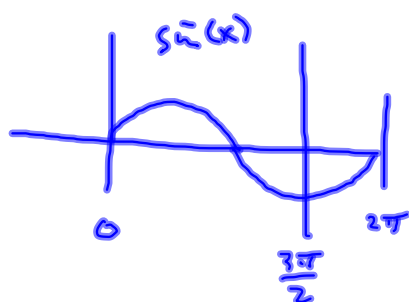


$$\int_0^4 (2x+3) dx = \frac{1}{2} \left[\frac{(2x+3)^2}{2} \right]_0^4 = \frac{1}{4} [121 - 9] = \frac{112}{4} = 28$$

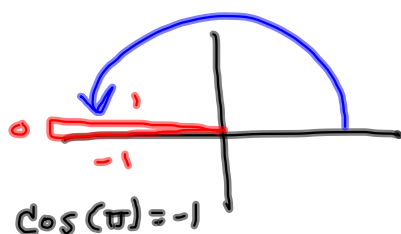
$A = \frac{1}{2} (B_1 + B_2) h$
 $= \frac{1}{2} (11 + 3) 4 = 28$
 $\triangle + \square$
 $= \frac{1}{2} b h + l w$
 $= \frac{1}{2} (4) (8) + 4 (3)$
 $= 16 + 12 = 28$

$$= \int_0^4 (2x+3) dx = \left[x^2 + 3x \right]_0^4 = 16 + 12 = 28$$

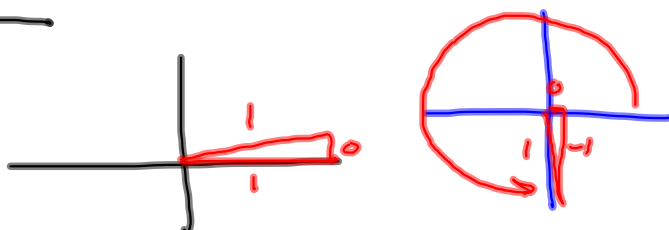
$$\int_0^{\frac{3\pi}{2}} |\sin(x)| dx = \int_0^{\pi} \sin(x) dx + \int_{\pi}^{\frac{3\pi}{2}} -\sin(x) dx$$



$$= -\cos(x) \Big|_0^{\pi} + \cos(x) \Big|_{\pi}^{\frac{3\pi}{2}} = -\cos(\pi) - (-\cos(0)) + \cos\left(\frac{3\pi}{2}\right) - \cos(\pi)$$



$$= -(-1) - (-1) + 0 - (-1) = 3$$



$$G(x) = \int_1^x \sin^2(\pi t) dt$$

$$\frac{dG}{dx} = \sin^2(\pi x)$$

$$\frac{d}{dx} \int_1^{\sin x} (t^2 - 7t) dt = ((\sin x)^2 - 7\sin x) \cos x$$

$$G(x^2+5) = \int_1^{x^2+5} \sin^2(\pi t) dt = \int_1^u \sin^2(\pi t) dt = G(u)$$

$$\frac{dG}{dx} = \frac{dG}{du} \cdot \frac{du}{dx} = \sin^2(\pi u) \cdot 2x = \sin^2(\pi(x^2+5)) \cdot 2x$$

$$\int_2^5 x^2 dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$$

$$x_k = a + k\Delta x = 2 + k \cdot \frac{3}{n} = 2 + \frac{3k}{n}$$

$$Area \approx \sum_{k=1}^n f(x_k) \Delta x = \Delta x \sum_{k=1}^n f(x_k) = \frac{3}{n} \sum_{k=1}^n f\left(2 + \frac{3k}{n}\right)$$

$$= \frac{3}{n} \sum_{k=1}^n \left(2 + \frac{3k}{n}\right)^2 = \frac{3}{n} \sum_{k=1}^n \left(4 + \frac{12k}{n} + \frac{9k^2}{n^2}\right)$$

$$\left(2 + \frac{3k}{n}\right)^2 = \left(2 + \frac{3k}{n}\right)\left(2 + \frac{3k}{n}\right) = 4 + \frac{6k}{n} + \frac{6k}{n} + \frac{9k^2}{n^2}$$

$$(a+b)^2 = a^2 + 2ab + b^2 \quad = 4 + \frac{12k}{n} + \frac{9k^2}{n^2}$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$