Area under $f(x) = x^2$ from $x = 2$ to $x = 5$

$[a,b] = [2,5]$, $\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n} = \Delta x$

$x_k = a + k\Delta x = 2 + k \cdot \frac{3}{n} = 2 + \frac{3}{n}k = x_k$

$$\text{Area} \approx \sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(2+\frac{3}{n}k\right)^2$$

$\Delta x_k = \frac{3}{n}$ for every k.

The most rigorous/general development of the theory lets Δx_k be different for each k, so pulling it outside doesn't work, depending on textbook treatment (degree of generality).

We'll always be able to pull out the Δx, because we make our rectangles the same width.

$$= \frac{3}{n}\sum_{k=1}^{n}\left(4 + \frac{12}{n}k + \frac{9}{n^2}k^2\right) = \frac{3}{n}\sum_{k=1}^{n} 4 + \frac{3}{n}\sum_{k=1}^{n}\frac{12}{n}k + \frac{3}{n}\sum_{k=1}^{n}\frac{9}{n^2}k^2$$

$$= \frac{3}{n}\cdot 4n + \frac{3}{n}\cdot\frac{12}{n}\cdot\left(\frac{n^2 + \text{lesser degree}}{2}\right) + \frac{3}{n}\cdot\frac{9}{n^2}\left(\frac{n^3 + \text{lesser degree}}{3}\right)$$

$$= 12 + \frac{18\,(n^2 + \text{lesser})}{n^2} + \frac{9\,(n^3 + \text{lesser degree})}{n^3}$$

$$\xrightarrow{n \to \infty} 12 + 18 + 9 = 39$$

People are making this boo-boo:

$$\frac{3}{n} \lim_{n\to\infty} \sum_{k=1}^{n} \left(2+\frac{3}{n}k\right)^2$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n}\left(4+\frac{12}{n}k+\frac{9}{n^2}k^2\right) = \lim_{n\to\infty}\left[\frac{3}{n}\left(4n+\frac{12}{n}\left(\frac{n^2+\text{small}}{2}\right) + \frac{9}{n^2}\cdot\left(\frac{n^3+\text{small}}{3}\right)\right)\right]$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}(4n) + \frac{36}{n^2}\left(\frac{n^2+\text{small}}{2}\right) + \frac{27}{n^3}\left(\frac{n^3+\text{small}}{3}\right)\right)$$

$$\xrightarrow{n\to\infty}$$

No.

$$= 12 + 18 + 9$$

20 $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left(5+\frac{2k}{n}\right)^{10}$

$\boxed{[a,b]=[5,7] \quad f(x)=x^{10}}$

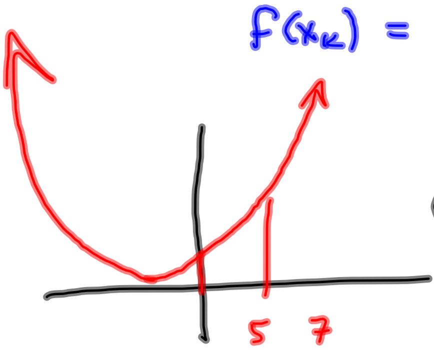
$\Delta x = \frac{2}{n} = \frac{b-a}{n}$

$x_k = a + k\Delta x = \boxed{5+\frac{2k}{n} = a + k\cdot\frac{2}{n}}$

$a=5$ & $\frac{b-a}{n} = \frac{2}{n} \Rightarrow b-a=2 \Rightarrow b=7$

$f(x_k) = x_k^{10}$

The region under $f(x)=x^{10}$ from $x=5$ to $x=7$



$(x+5)^{10}$ on $[0,2]$

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

$$\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \sin^{-2}(x)\cos(x)\,dx$$

$$\left(u = \sin(x) \Longrightarrow du = \cos(x)\,dx \Longrightarrow \frac{du}{\cos(x)} = dx \right)$$

$$= \int u^{-2}\cos(x)\cdot\frac{du}{\cos(x)} = \int u^{-2}\,du = \frac{1}{-1}u^{-1} + C$$

$$= -\sin^{-1}(x) + C$$

$$= -\frac{1}{\sin(x)} + C$$

$$= -\csc(x) + C$$

$$\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx = \int \cos\left(\frac{\pi}{x}\right)\cdot\frac{1}{x^2}\,dx$$

$$= \int \cos\left(\pi x^{-1}\right)\cdot x^{-2}\,dx \quad = \int \cos(u)\cdot x^{-2}\cdot\left(\frac{-x^2\,du}{\pi}\right)$$

$$\left(u = \pi x^{-1} \Longrightarrow du = -\pi x^{-2}\,dx \quad \frac{x^2\,du}{-\pi}\right)$$

$$\left(x^{-2}\right)\left(x^{2}\right) = x^{-2+2} = x^0 = 1$$

$$= -\frac{1}{\pi}\int \cos(u)\,du$$