$$\int_1^2 x\sqrt{x-1}\,dx \qquad \int_{x=1}^{2=x} \longrightarrow \int_{u=0}^{u=1}$$

$$\boxed{u = x-1} \Longrightarrow u+1 = x$$

$$du = dx$$

$$\int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u+1)\,u^{\frac{1}{2}}\,du = \int_0^1 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right]_0^1 = \frac{2}{5} + \frac{2}{3} = \frac{6+10}{15} = \frac{16}{15}$$

$$\int_1^2 x\sqrt{x-1}\,dx$$

one that takes a u-substitution that does NOT involve reconstructing the integrand as a derivative resulting from application of chain rule.

$$\int (u^2+1)\,u\,du$$

$u=\sqrt{x-1}$

$u^2=x-1$

$\boxed{u^2+1=x}$

$du=\frac{1}{2}(x-1)^{-\frac{1}{2}}dx$

$dx=2\sqrt{x-1}\,du$

$$\int_{0=u}^{1=u}(u^2+1)\cdot u\,2\sqrt{x-1}\,du=\int_0^1(u^2+1)\cdot u\cdot 2\cdot u\,du=2\int_0^1 u^2(u^2+1)\,du$$

$$=2\int_0^1(u^4+u^2)\,du$$

$u=\sqrt{x-1}$

$$\boxed{\begin{array}{l} x=1 \Rightarrow u=0 \\ x=2 \Rightarrow u=1 \end{array}}$$

$$=2\left[\frac{1}{5}u^5+\frac{1}{3}u^3\right]_0^1$$

$$=2\left[\frac{1}{5}+\frac{1}{3}\right]=2\left[\frac{3+5}{15}\right]=2\left[\frac{8}{15}\right]=\frac{16}{15}$$

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④ $\int \frac{x}{\sqrt{a+bx}}\,dx$ $\qquad (fg)' = f'g + fg'$

$$= \frac{2}{3b^2}(bx-2a)\sqrt{a+bx} + C$$

$$\frac{d}{dx}\left[\frac{2}{3b^2}\underline{(bx-2a)(a+bx)^{\frac{1}{2}}} + c\right]$$

$$= \frac{2}{3b^2}\left[b(a+bx)^{\frac{1}{2}} + (bx-2a)\left(\tfrac{1}{2}\right)(a+bx)^{-\frac{1}{2}}(b)\right]$$

$$= \frac{2}{3b^2}\left[\frac{b\sqrt{a+bx}}{1}\cdot\frac{2\sqrt{a+bx}}{2\sqrt{a+bx}} + \frac{b(bx-2a)}{2\sqrt{a+bx}}\right]$$

$$= \frac{2}{3b^2}\left[\frac{2b(a+bx)+b^2x-2ab}{2\sqrt{a+bx}}\right]$$

$$= \frac{2}{3b^2}\left[\frac{2ab+2b^2x+b^2x-2ab}{2\sqrt{a+bx}}\right] = \frac{2}{3b^2}\left[\frac{3b^2x}{2\sqrt{a+bx}}\right]$$

$$= \frac{x}{\sqrt{a+bx}} \quad ✓$$

$$\int_3^5 f(x)\,dx \approx \sum \frac{ft}{mi}\cdot mi = ft + ft + ft + \cdots + ft$$

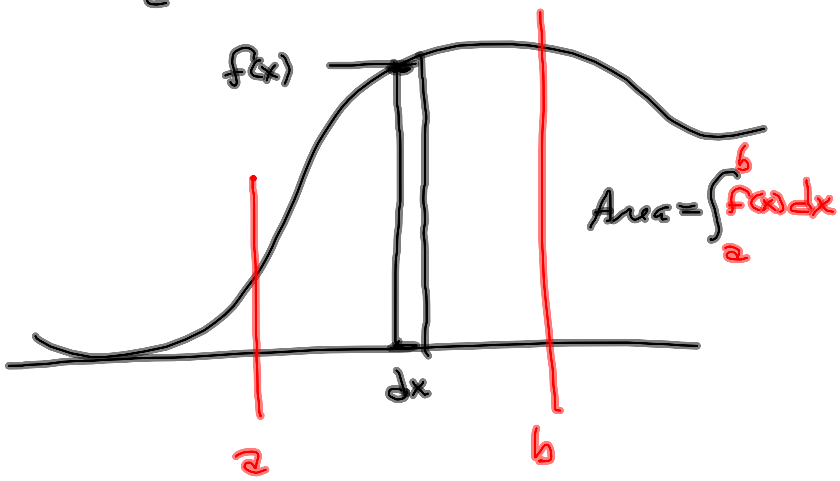
Say slope is in ft/mile

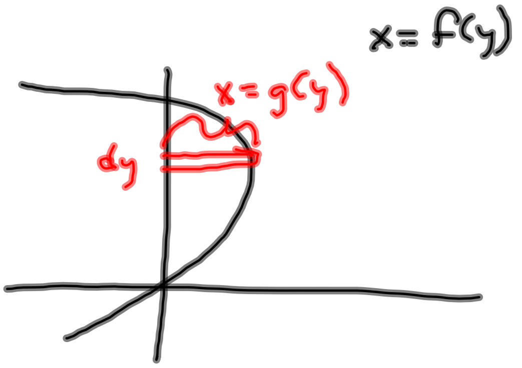
$$\int \frac{\text{vertical change}}{\boxed{\text{unit change in } x}} \cdot \text{miles} = \text{feet}$$

measured in miles

d $\dfrac{\Delta y}{\Delta x} = \dfrac{\text{rise}}{\text{run}} \approx \dfrac{dy}{dx}$

$$\int_a^b f(x)\,dx \approx \sum f(x_k)\,\Delta x$$





$$\frac{x^3 \sin x}{\cos x + 1x^0}$$

$f(-x) = f(x)$ $+$

$\frac{(-)(-)}{+} = +$ $f(-x) = -f(x)$ $-$

even ± even = even
odd ± odd = odd

$-(-(-4)) = -4$

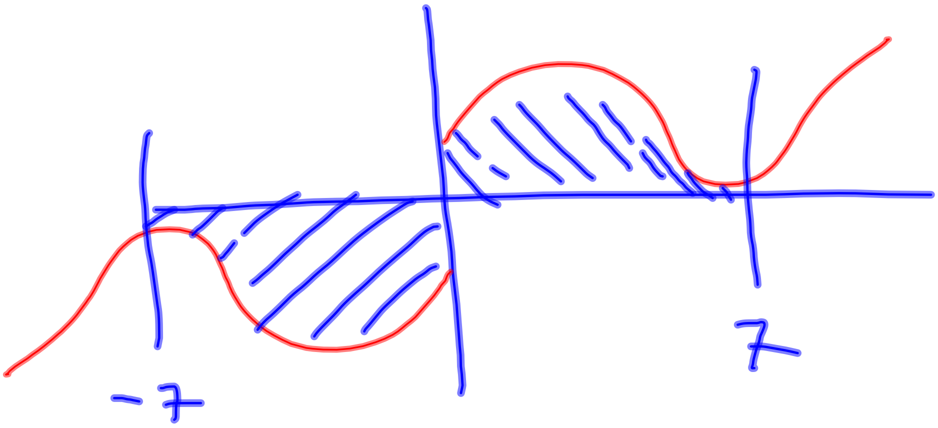
Think of odd funcs as "$-$"
" " even funcs as "$+$"

$$\frac{x^4}{x^3} - \frac{5x^3}{\cos x}$$

$\frac{+}{(-)(+)} = -$

odd

$$\int_{-7}^{7} \frac{x^2}{2} \sin x \cos x \, dx = 0$$



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$$\boxed{\frac{d}{dx}\int_0^{3x+2} t^2\cos(3t)\,dt = 3\cdot(3x+2)^2\cos(3(3x+2))}$$

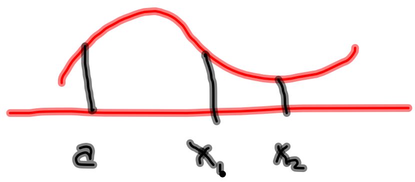
FTC I w/
chain Rule

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\,u'(x)$$

$$G(x) = \int_a^x f(t)\,dt$$

$$G'(x) = f(x)$$

$$\frac{dG}{dx} = f(x)$$



chain Rule

$$G(u(x)) = \int_a^{u(x)} f(t)\,dt$$

$$\frac{d}{dx}\left[G(u(x))\right] =$$

$$\frac{dG}{du}\cdot\frac{du}{dx} = G'(u)\,u'(x)$$

$$\frac{d}{dx}\left[(3x+2)^5\right] = \underbrace{\frac{d}{d(3x+2)}\left[(3x+2)^5\right]} \cdot \frac{d}{dx}\left[3x+2\right]$$

$$= 5(3x+2)^4 \cdot 3$$

$$= \frac{dG}{du}\cdot\frac{du}{dx}$$