36 5.3

$$\int_{-2}^{2} f(x)\,dx$$

$$f(x) = \begin{cases} 2 & -2 \le x \le 0 \\ 4-x^2 & \text{if } 0 < x \le 2 \end{cases}$$

5.2

#36

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx$$

can't be evaluated w/ current techniques

$y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2+y^2=4$

top $\frac{1}{2}$ of circle, centered @ (0,0) with $r=2$

$\frac{1}{2}\pi(2)^2 = 2\pi$

Apr 11-12:07 PM

§5.4 $\int_{t=a}^{t=b} \text{velocity}(t)\,dt = \text{Net Distance.}$

(Any time $v(t)$ is negative, those distances will subtract.)

$$\int_a^b \text{rate} = \text{Net change.}$$

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\int_a^b F'(t)\,dt = \int_1^2 v(t)\,dt = \int_1^2 \underset{(ft/s)}{(t^2-3t)}\underset{(s)}{dt}$$ is in feet

(v = velocity, as a function of (in ft/s)
t = time (in s))

$$\left[\frac{1}{3}t^3 - \frac{3}{2}t^2\right]_1^2 = \underbrace{\frac{1}{3}(2)^3 - \frac{3}{2}(2)^2}_{F(b)} - \underbrace{\left[\frac{1}{3}(1)^3 - \frac{3}{2}(1)^2\right]}_{F(a)}$$

$$= \frac{8}{3} - 6 - \frac{1}{3} + \frac{3}{2}$$

$$= \frac{7}{3} + \frac{3}{2} - 6$$

$$= \frac{14+9-36}{6} = \boxed{\frac{-13}{6}\text{ ft}}$$

Engage w/ 5.4 & ask on Tuesday

5.4 due Wed.

§5.5 u-substitution.

$2(x^2+5)(2x)$ looks like chain rule being applied to $(x^2+5)^2$

$u = x^2+5$

$\frac{du}{dx} = 2x$

$$\int 2(x^2+5)(2x)\,dx = (x^2+5)^2 + C$$

One way of looking at it:

Key Step → $\int 2u \cdot \frac{du}{\cancel{dx}} \cdot \cancel{dx} = \int 2u\,du = 2\cdot\frac{u^2}{2} + C = u^2 + C$

$= (x^2+5) + C$

The trick is seeing u and its derivative.

$$\int f'(u)\cdot \frac{du}{dx}\cdot dx = \int f'(u)\,du = f(u) + C$$

$$\int x(x^2+5)\,dx = \int x\cdot u\cdot \frac{du}{2x} = \int \frac{u}{2}du = \frac{1}{2}\int u\,du$$

If $u = x^2+5$

Then $\frac{du}{dx} = 2x$

and so $du = 2x\,dx$

thus, $dx = \frac{du}{2x}$

$$= \frac{1}{2}\frac{u^2}{2} + \frac{c}{2} = \frac{u^2}{4} + C$$

$$= \frac{(x^2+5)^2}{4} + C$$

Check:

$$\frac{2(x^2+5)(2x)}{4} + 0$$

$$= x(x^2+5)$$

$$\int (x^2+5)^{37}\,dx = \int u^{37} \cdot \frac{du}{2x}$$

There's no x to cancel with THIS x.

$u = x^2+5$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

But where's the x in the integrand?

This technique breaks down if you can't generate du just by multiplying or dividing by a constant.

$$\int \sin(\pi x)\,dx = \int \sin(u)\,\frac{du}{\pi} = \frac{1}{\pi}\int \sin(u)\,du$$

$$\left.\begin{aligned} u &= \pi x \\ du &= \pi\,dx \\ \frac{du}{\pi} &= dx \end{aligned}\right\} \text{Book way}$$

$$= \frac{1}{\pi}\left(-\cos(u)\right) + C$$

$$= -\frac{1}{\pi}\cos(\pi x) + C$$

$$\frac{d}{dx}\left[-\frac{1}{\pi}\cos(\pi x) + C\right]$$

$$= -\frac{1}{\pi}\left(-\sin(\pi x)\cdot\pi\right) = \sin(\pi x) \checkmark$$

$$\int \sin(x^2+3x)\,dx$$

$$u = x^2 + 3x$$

$$du = \boxed{(2x+3)}\,dx$$

Nothing to build this with.

$$\int \sin(x^2+3x)\,dx$$

Can't do it.

$u = x^2+3x$

$du = \boxed{(2x+3)}\,dx$

Nothing to build this with.

$$\int \sin(x^2+3x)\cdot(2x+3)\,dx$$

$u = x^2+3x$

$du = (2x+3)dx$

$$= \int \sin(u)\,du$$

etc.

We <u>can</u> do this one, thanks to the derivative of u living here.

$$\int_0^{\frac{\pi}{2}} x\cos(x^2)\,dx$$

(here's how I do 'em:
$u = x^2 \Longrightarrow du = 2x\,dx$)

$$= \frac{1}{2}\int \cos(x^2)\cdot 2x\,dx$$

Book approach

$u = x^2 \Longrightarrow du = 2x\,dx \Longrightarrow dx = \frac{du}{2x}$

$$\int_{0=x}^{\frac{\pi}{2}=x} x\cos(x^2)dx = \int x\cdot\cos(u)\cdot\frac{du}{2x} = \frac{1}{2}\int_{0=x}^{\frac{\pi}{2}=x}\cos(u)\,du$$

Need to convert to u's

$u = x^2 \Longrightarrow$

$x = 0 \Longrightarrow u = 0^2$ &

$x = \frac{\pi}{2} \Longrightarrow u = \frac{\pi^2}{4}$

$$= \frac{1}{2}\int_0^{\frac{\pi^2}{4}}\cos(u)\,du$$

$$= \left[\frac{1}{2}\sin(u)\right]_0^{\frac{\pi^2}{4}} = \boxed{\frac{1}{2}\sin\left(\frac{\pi^2}{4}\right)}$$

$$\int_{0=x}^{\frac{\pi}{2}=x} x\cos(x^2)\,dx =$$

one way to sidestep the change of variables in the upper & lower limits of integration. Just do the indefinite integral and then plug into the antiderivative.

$$\int x\cos(x^2)\,dx = \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + C$$

$$= \frac{1}{2}\sin(x^2) + C.$$

So, $\displaystyle\int_0^{\frac{\pi}{2}} x\cos(x^2)\,dx = \left[\frac{1}{2}\sin(x^2)\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\sin\left(\left(\frac{\pi}{2}\right)^2\right)$ = same as before.

$$\int_1^2 x\sqrt{x-1}\,dx \qquad \int_{x=1}^{2=x} \longrightarrow \int_{u=0}^{u=1}$$

$u = x-1 \Longrightarrow u+1 = x$

$du = dx$

$$\int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u+1)u^{\frac{1}{2}}\,du = \int_0^1 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$$

$u = \sqrt{x-1}$

$u^2 = x-1$

$u^2+1 = x$

$$\int (u^2+1)u$$

$du = \frac{1}{2}(x-1)^{-\frac{1}{2}}dx$

$dx = 2\sqrt{x-1}\,du$

$$\int (u^2+1)\cdot u \, 2\sqrt{x-1}\,du$$

what to do?

$u = \sqrt{x-1}$

$x=1 \Rightarrow u=0$

$x=2 \Rightarrow u=1$

$$\int (u^2+1)(\sqrt{x-1})(2\sqrt{x-1})\,du$$

$$= 2\int_0^1 (u^2+1)(u^2)\,du$$