See next page for the sequel.

$$\sum_{k=1}^{3} 5k = 5(1) + 5(2) + 5(3) = 5(1+2+3)$$

$$= 5\sum_{k=1}^{3} k$$

K stays in.
Anything else factors out.

$$\boxed{\sum c f(k) = c \sum f(k)}$$

$$\boxed{\sum_{k=1}^{3} (f(k) + g(k))} = f(1) + g(1) + f(2) + g(2) + f(3) + g(3)$$

$$= f(1) + f(2) + f(3) + g(1) + g(2) + g(3)$$

$$\boxed{= \sum_{k=1}^{3} f(k) + \sum_{k=1}^{3} g(k)}$$

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

from $\sum (a+b) = \sum a + \sum b$

$$\int c f(x)\,dx = c \int f(x)\,dx$$

from $\sum c f(k) = c \sum f(k)$

Something like §5.1 #22

Area under $f(x) = x^4 \quad 0 \le x \le 2$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

$$x_k = a + k\Delta x = 0 + \frac{2}{n}\cdot k = \frac{2}{n}k$$

$$\text{Area} = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n}k\right)^4 \cdot \frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2^4}{n^4}k^4 \cdot \frac{2}{n} =$$

$an^4 + bn^3 + cn^2 + dn + e$ vanishes in the limit, because of the n^5 downstairs to extinguish it

$$= \lim_{n\to\infty} \frac{32}{n^5} \sum_{k=1}^{n} k^4 = \lim_{n\to\infty} \frac{32}{n^5}\left[\frac{n^5 + \text{smaller}}{5}\right] = \frac{32}{5}$$

$$\lim_{x\to\infty} \frac{5x^7 \underbrace{-3x+2}_{\text{smaller}}}{\underbrace{3}_{\text{smaller}} - 2x^7} = \lim_{x\to\infty} \frac{5x^7}{-2x^7} = -\frac{5}{2}$$

$$\int_0^1 x^3\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{n}k\right)^3 \cdot \frac{1}{n} =$$

$$\lim_{n\to\infty} \frac{1}{n^4} \sum_{k=1}^{n} k^3 = \lim_{n\to\infty} \frac{1}{n^4}\left[\frac{n^4}{4} + \underbrace{\frac{\text{smaller}}{4}}_{\to 0}\right]$$

$$= \frac{1}{4}$$

$$\sum_{k=1}^{n} \left(\frac{1}{n}k\right)^3 \cdot \frac{1}{n} = \frac{1}{n^4} \sum_{k=1}^{n} k^3 = \frac{1}{n^4}\left[\frac{n^4 + \text{smaller degree}}{4}\right]$$

$$\frac{n^4}{4n^4} + \frac{\text{small}}{4n^4} \xrightarrow{n\to\infty} \frac{1}{4}$$

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§5.3

FTC : f is continuous on $[a,b]$

Then $\displaystyle\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b)-F(a)$, where

F is ANY antiderivative of f.

(Choose F to be the one where $C=0$)

A quick formality:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Formally, this means that, given $\varepsilon > 0$, there is an $N > 0$ such that

$$\left| \boxed{\int_a^b f(x)\,dx} - \sum_{k=1}^{n} f(x_k)\Delta x \right| < \varepsilon$$

for every $n > N$.

We come as close as we want to $\int_a^b f(x)\,dx$ by making n as big as we need.

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$$\int_0^1 x^3\,dx = \frac{x^4}{4}\Big]_0^1 = \frac{1^4}{4} - \frac{0^4}{4} = \frac{1}{4}$$

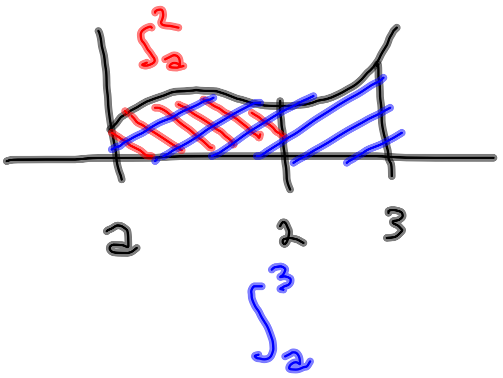
$$\left[\frac{x^4}{4} + 7\right]_0^1 = \frac{1^4}{4} + 7 - \left[\frac{0^4}{4} + 7\right]$$
$$= \frac{1}{4}$$

Suppose $f(x)$ is positive.

Then $\int_a^b f(t)\,dt$ is a positive number.

and $\int_a^x f(t)\,dt$ is (an increasing) function of x!

$$\int_a^2 f(t)\,dt \le \int_a^3 f(t)\,dt = \int_a^2 f(t)\,dt + \int_2^3 f(t)\,dt$$



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$$\int_1^x t\,dt = \frac{t^2}{2}\Big]_1^x = \frac{x^2}{2} - \frac{1}{2}$$

$$\frac{d}{dx}\left[\frac{x^2}{2} - \frac{1}{2}\right] = x$$. So,

$$\frac{d}{dx}\int_1^x t\,dt = x$$

$$\frac{d}{dx}\int_1^x t^2\,dt = \frac{d}{dx}\left[\left[\frac{t^3}{3}\right]_1^x\right] = \frac{d}{dx}\left[\frac{x^3}{3} - \frac{1}{3}\right] = x^2$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

what about $\frac{d}{dx}\int_0^{\sin x} t^2\,dt$?

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$$\int_0^{\sin x} t^2\,dt = \frac{t^3}{3}\Big]_0^{\sin x} = \frac{\sin^3 x}{3} - \frac{0^3}{3} = \frac{1}{3}\sin^3 x$$

and $\frac{d}{dx}\left[\frac{1}{3}\sin^3(x)\right] = \frac{1}{3}\frac{d}{dx}\left[\sin^3(x)\right]$

$(\sin(x))^3$ $= \frac{1}{3}\cdot 3\sin^2 x \cdot \cos x$

$= \sin^2 x \cos x$

by chain rule.

$$\frac{d}{dx}\int_0^{\sin x} t^2\,dt = \sin^2(x)\cdot\cos x$$

$$\boxed{\frac{d}{dx}\int_a^{u(x)} f(t)\,dt = f(u(x))\cdot u'(x)}$$

Chain Rule Version of FTC I

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$$\frac{d}{du}\int_a^x f(t)\,dt = 0$$

$$\frac{d}{du}\int_a^x f(u)\,du = \frac{d}{du}\int_a^x f(z)\,dz = 0$$

Just a dummy variable, like k, in

$$\sum_{k=1}^{50} f(x_k)$$

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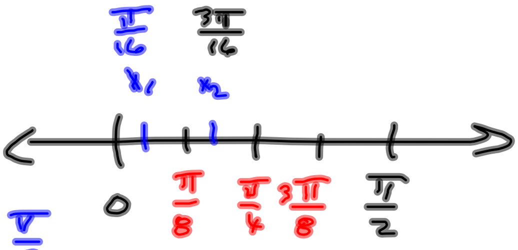
§5.2 #10

$$\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$$



$$\Delta x = \frac{\frac{\pi}{2} - 0}{n} = \frac{\pi}{2n} = \frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$$

$$x_k = a + k\Delta x = 0 + \frac{\pi}{8}k$$

$$\sum_{k=1}^{4} f(x_k)\Delta x = \Delta x \sum_{k=1}^{4} f(x_k)$$

$$= \frac{\pi}{8}\left[\underbrace{\cos^4\left(\frac{\pi}{16}\right) + \cdots + \cos^4(x_4)}_{\text{Do this 1st.}}\right]$$