§5.1 #18

$f(x) = 1 + x^4, \quad 2 \le x \le 5$
on $[2,5]$

$$\boxed{\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x}$$

$\frac{b-a}{n} = \Delta x = \frac{5-2}{n} = \frac{3}{n} = \Delta x$

Need $x_k = a + k\Delta x$ for right endpoints (Easiest)

$a=2,\ b=5,\ \Delta x = \frac{3}{n}$ = mesh of the partition.

$a + k\Delta x = 2 + k\cdot\frac{3}{n} = 2 + \frac{3}{n}k$

is an upper estimate (Right endpoints, $f(x)$ is increasing)

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[(1 + x_k^4)\cdot\frac{3}{n}\right] = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(1 + \left(2 + \frac{3}{n}k\right)^4\right)\cdot\frac{3}{n}\right]$$

To evaluate this, we'd need

$\sum_{k=1}^{n} 1 = n, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2}{2} + \frac{n}{2} = \frac{n^2}{2} + \text{sm}\,n$

$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \text{smaller}, \quad \sum_{k=1}^{n} k^3 = \frac{n^4}{4} + \text{smaller},$

$\sum_{k=1}^{n} k^4 = \frac{n^5}{5} + \text{smaller}$

FYI : Midpoint : $x_k = a + \frac{1}{2}\Delta x + (k-1)\Delta x$

#5 $x_1 = -\frac{3}{4}$ $n=6$ $x_1 = a + \frac{1}{2}\Delta x$

$x_2 = -\frac{1}{4}$ $\Delta x = \frac{1}{2}$ $x_2 = a + \frac{1}{2}\Delta x + \Delta x$

$x_3 = \frac{1}{4}$ $x_3 = a + \frac{1}{2}\Delta x + 2\Delta x$

Left Endpoint: $x_k = a + (k-1)\Delta x$

In practice, it's easier to just look at the picture/number line.

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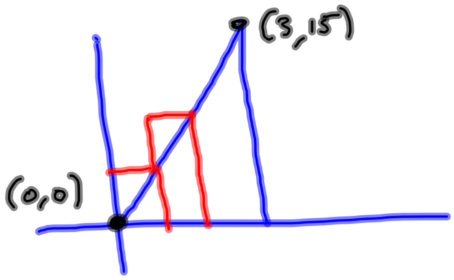
Use the definition of area to find the area under $f(x) = 5x$, for $0 \le x \le 3$



$\frac{1}{2} \cdot 3 \cdot 15 = \boxed{\frac{45}{2}}$

Use Calculus:

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

$x_k = 0 + k\Delta x = k \cdot \frac{3}{n} = \frac{3}{n}k$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} 5x_k \Delta x = \sum_{k=1}^{n} 5\left(\frac{3}{n}k\right) \cdot \frac{3}{n}$$

$$= \frac{9}{n^2}\sum_{k=1}^{n} 5k = \frac{45}{n^2}\sum_{k=1}^{n} k = \frac{45}{n^2}\left[\frac{n^2}{2} + \frac{n}{2}\right]$$

$$\frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$= \frac{45n^2}{2n^2} + \frac{45n}{2n^2} = \frac{45}{2} + \frac{45}{2n}$$

$$\xrightarrow{n \to \infty} \boxed{\frac{45}{2}} + 0$$

The smaller stuff vanishes in the limit!

$$\sum_{k=1}^{3} 5k = 5(1) + 5(2) + 5(3) = 5(1+2+3)$$

$$= 5\sum_{k=1}^{3} k$$

K stays in.
Anything else
factors out.

$$\boxed{\sum c\,f(k) = c\sum f(k)}$$

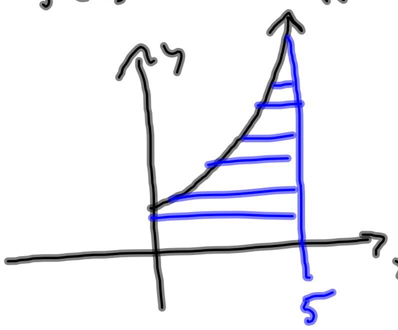
$$\boxed{\sum_{k=1}^{3}\left(f(k)+g(k)\right)} = f(1)+g(1)+f(2)+g(2)+f(3)+g(3)$$

$$= f(1)+f(2)+f(3)+g(1)+g(2)+g(3)$$

$$\boxed{= \sum_{k=1}^{3} f(k) + \sum_{k=1}^{3} g(k)}$$

Find the exact area under

$f(x) = x^2 + 3x + 1$, for $0 \le x \le 5$



$\Delta x = \frac{5}{n}$

$x_k = a + k\Delta x = 0 + \frac{5}{n}k = \frac{5}{n}k$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left[\left(\left(\frac{5}{n}k\right)^2 + 3\left(\frac{5}{n}k\right) + 1\right)\cdot \frac{5}{n}\right]$$

$$= \sum_{k=1}^{n} \left(\frac{25}{n^2}k^2 + \frac{15}{n}k + 1\right)\frac{5}{n}$$

$$= \frac{5}{n}\left[\sum_{k=1}^{n} \frac{25}{n^2}k^2 + \sum_{k=1}^{n} \frac{15}{n}k + \sum_{k=1}^{n} 1\right]$$

$$= \frac{5}{n}\sum_{k=1}^{n} \frac{25}{n^2}k^2 + \frac{5}{n}\sum_{k=1}^{n} \frac{15}{n}k + \frac{5}{n}\sum_{k=1}^{n} 1$$

$$= \frac{125}{n^3}\sum_{k=1}^{n} k^2 + \frac{75}{n^2}\sum_{k=1}^{n} k + \frac{5}{n}\sum_{k=1}^{n} 1$$

$$= \frac{125}{n^3}\cdot\frac{n^3 + \text{smaller degree}}{3}$$

Just like

$$\lim_{x\to\infty} \frac{5x^3+2x^2-1}{7x^3-5x} = \frac{5}{7}$$

$$+ \frac{75}{n^2}\cdot\frac{n^2 + \text{smaller degree}}{2}$$

$$+ \frac{5}{n}\cdot n \xrightarrow{n\to\infty} \frac{125}{3} + \frac{75}{2} + 5 = \frac{250+225+30}{6}$$

$$= \frac{505}{6}$$

$$\int_0^5 (x^2+3x+1)\,dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} + x\right]_0^5$$

$$= \frac{5^3}{3} + \frac{3(5)^2}{2} + 5 - \left(\frac{0^3}{3} + \frac{3\cdot 0^2}{2} + 0\right)$$

$$= \frac{125}{3} + \frac{75}{2} + 5$$

5.3

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d}{dx}\overbrace{\int_a^x f(t)\,dt}^{G(x)} = f(x) \qquad G'(x)$$

FTC I

$$G(x^3) = G(u(x))$$

$$\frac{d}{dx}\overbrace{\int_{212060}^{x^3} f(t)\,dt}^{} = f(x^3)\cdot 3x^2 \qquad \frac{dG}{du}\cdot\frac{du}{dx}$$

Chain Rule for FTC I

$$\frac{d}{dx}\left[(x^3+1)^2\right] = 2(x^3+1)'(3x^2)$$

$$G(x) = x^2 \qquad \frac{dG}{du} = 2u^2$$

$$u(x) = x^3+1 \qquad \frac{du}{dx} = 3x^2$$

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