§5.1 #s 5, 12, 17–20, 22

§5.2 #s 9, 10, 17, 18, 21, 22, 29, 36, 37

§5.3 #s $\underbrace{7, 8, 11, 13, 14,}_{\text{FTC I}}$ $\underbrace{19, 22, 23, 28, 32, 37, 38}_{\text{FTC II}}$

$$\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2} = \frac{n^2}{2} + \text{smaller stuff}$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3}{6} + \text{smaller stuff}$$

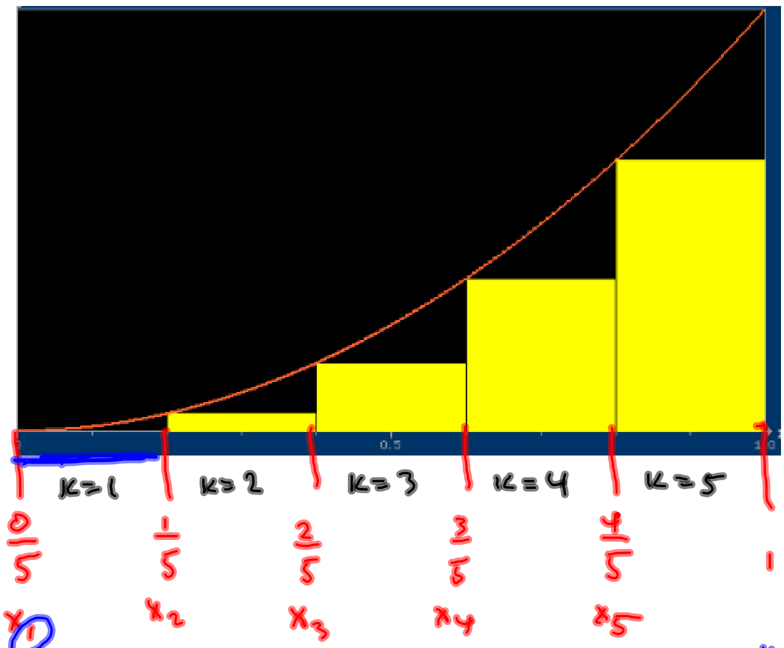
$$= \frac{1}{3}n^3 + \text{smaller}$$

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \cdots = \frac{n^4}{4} + \text{smaller}$$

$$\sum_{k=1}^{n} k^5 = \cdots = \frac{n^6}{6} + \text{smaller}$$

The area under $f(x) = x^2$ on $[0,1]$

Use left end points and 5 rectangles.



$0 = (1-1)(\frac{1}{5})$

$[a,b] = [0,1]$

Δx = width of each rectangle is $\frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5}$

The height of the k^{th} rectangle is...

$k=1,\ x_k = x_1 = 0$

$k=2,\ x_2 = \frac{1}{5} = 0 + (2-1)\cdot\frac{1}{5}$

$k=3,\ x_k = \frac{2}{5} = 0 + (3-1)\cdot\frac{1}{5}$

$k=4,\ x_k = \frac{3}{5} = 0 + (4-1)\cdot\frac{1}{5}$

In general:

Left Endpoints: $x_k = a + (k-1)\Delta x$

$$\boxed{= 0 + (k-1)\cdot\frac{1}{5}}$$

$\to x_k$

$$\text{Area} \approx \sum_{k=1}^{5} (\text{Area of Rectangle } k)$$

Left Endpoints: $x_k = a + (k-1)\Delta x$

$$\boxed{= 0 + (k-1)\cdot\frac{1}{5}} \to x_k$$

$$\text{Area} \approx \sum_{k=1}^{5} (\text{Area of Rectangle } k)$$

$$= \sum_{k=1}^{5} f(x_k)\Delta x = \sum_{k=1}^{5} x_k^2 \Delta x = \sum_{k=1}^{5} \left((k-1)\cdot\frac{1}{5}\right)^2 \cdot \frac{1}{5}$$

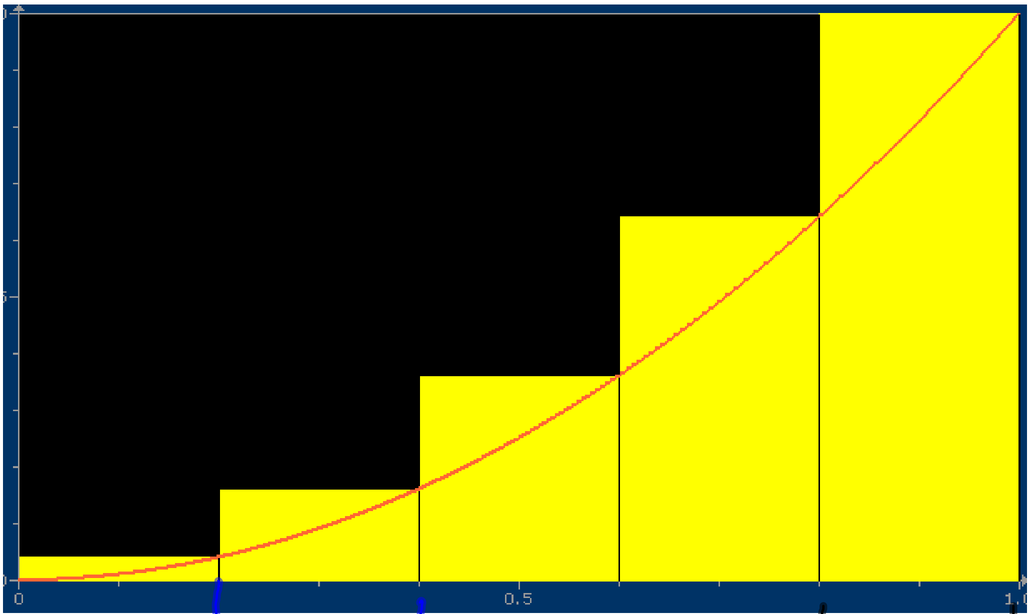
$$= \sum_{k=1}^{5} \left(\frac{k-1}{5}\right)^2 \cdot \frac{1}{5} = \sum_{k=1}^{5} \frac{k^2-2k+1}{25}\cdot\frac{1}{5} = \frac{1}{125}\sum_{k=1}^{5}(k^2-2k+1)$$

$$= \frac{1}{125}\Big[1^2-2(1)+1 + 2^2-2(2)+1 + 3^2-2(3)+1 + 4^2-2(4)+1 + 5^2-2(5)+1\Big]$$

$$= \frac{1}{125}\Big[0+1+4+9+16\Big] = \frac{30}{125} = \frac{6}{25}$$

$$= .24$$

Same question, with right endpoints.



x_1 $\quad x_2 = \frac{2}{5}$ $\quad x_3 = \frac{3}{5}$ $\quad \frac{4}{5}$ $\quad \frac{5}{5} = x_5$

$$x_1 = \frac{1}{5} = 0 + 1\cdot\frac{1}{5} = 0 + k\cdot\frac{1}{5} = \boxed{\frac{1}{5}k = x_k}$$

$$= a + k\cdot\Delta x = a + k\cdot\frac{b-a}{n} = x_k$$

for right endpoints.

Area of the k^{th} rectangle is

height · width

$= f(x_k) \cdot \frac{b-a}{n}$

$= x_k^2 \cdot \frac{1}{5}$

$= \left(\frac{1}{5}k\right)^2 \cdot \frac{1}{5}$

Area under the curve is

$A \approx \sum_{k=1}^{5} \left(\frac{1}{5}k\right)^2 \cdot \frac{1}{5} = \frac{1}{125} \sum_{k=1}^{n} k^2$

$= \frac{1}{125} \left[1^2 + 2^2 + 3^2 + 4^2 + 5^2\right] = \frac{1}{125} \cdot \left[1 + 4 + 9 + 16 + 25\right]$

$= \frac{1}{125} \cdot 55 = .44$

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How about Right endpoints for an arbitrary # of rectangles?

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n} = \text{width}$$

$$x_k = a + k \cdot \Delta x = a + k \cdot \frac{1}{n} = a + \frac{1}{n}k = a + \frac{k}{n}$$

$$x_1 = 0 + \frac{1}{n} \qquad \boxed{x_k = \frac{k}{n}}$$

$$x_2 = 0 + \frac{2}{n}$$

$$\vdots$$

$$f(x_k) = x_k^2 = \left(\frac{k}{n}\right)^2$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 \frac{1}{n}$$

$$= \sum_{k=1}^{n} \frac{k^2}{n^2} \cdot \frac{1}{n} = \sum_{k=1}^{n} \frac{k^2}{n^3} = \frac{1}{n^3}\left[\sum_{k=1}^{n} k^2\right]$$

$$= \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \text{verging on calculus} = \frac{1}{n^3}\left[\frac{1}{3}n^3 + \text{smaller}\right]$$

$$= \cancel{\frac{1}{n^3}} \cdot \frac{\cancel{n^3}}{3} + \cancel{\frac{1}{n^3} \cdot \frac{\text{smaller than degree 2}}{1}}$$

$$\xrightarrow{n \to \infty} \frac{1}{3}$$

$$\underbrace{\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = \frac{1}{3}}_{\text{FTC II}}$$