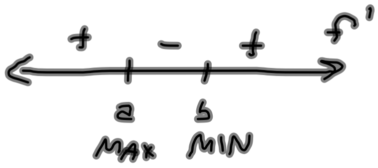
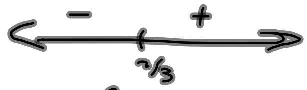
$f(x) = x^3 - 2x^2 - 5x + 10$

$f'(x) = 3x^2 - 4x - 5 \overset{SET}{=} 0 \Longrightarrow x = \frac{2 \pm \sqrt{19}}{3}$

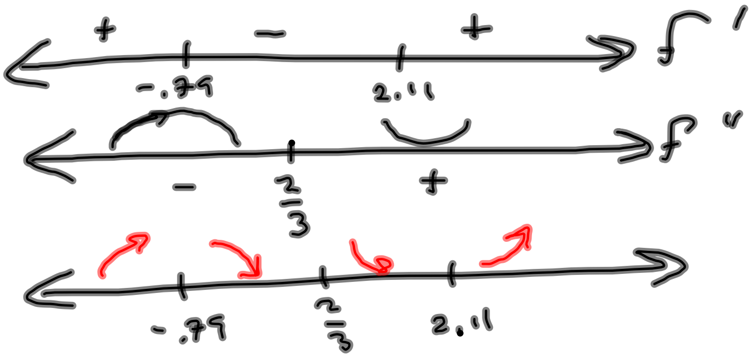


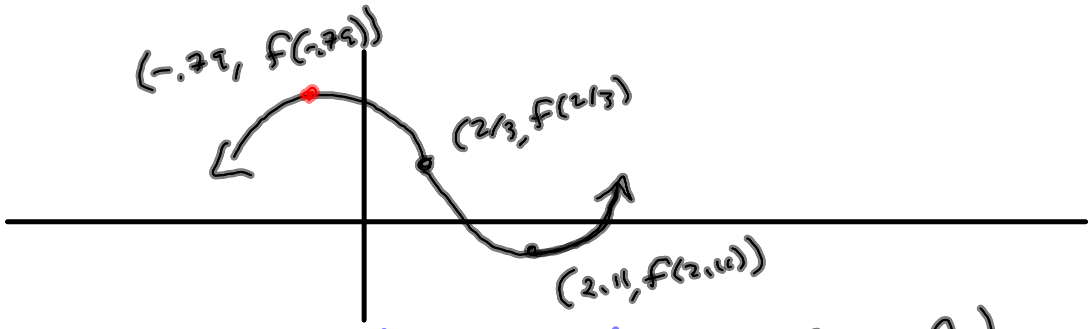
$f''(x) = 6x - 4 \overset{SET}{=} 0 \Longrightarrow x = \frac{2}{3}$

$\frac{2-\sqrt{19}}{3} \approx -.7863$ is neg



$\frac{2+\sqrt{19}}{3} \approx 2.1196$ is pos & bigger than $\frac{2}{3}$





x-intercepts done yesterday (clumsily)

Find local and absolute extremes
for $f(x) = x^3 - 6x^2 + 2$ on $[-1,2]$

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 0 & 2 \\ & & -1 & 7 & -7 \\ \hline & 1 & -7 & 7 & \underline{-5 = f(-1)} \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -6 & 0 & 2 \\ & & 2 & -8 & -16 \\ \hline & 1 & -4 & -8 & \underline{-14 = f(2)} \end{array}$$

$$f'(x) = 3x^2 - 12x \overset{SET}{=} 0 \implies 3x(x-4) = 0$$

$$\implies x = 0,\ x = 4$$

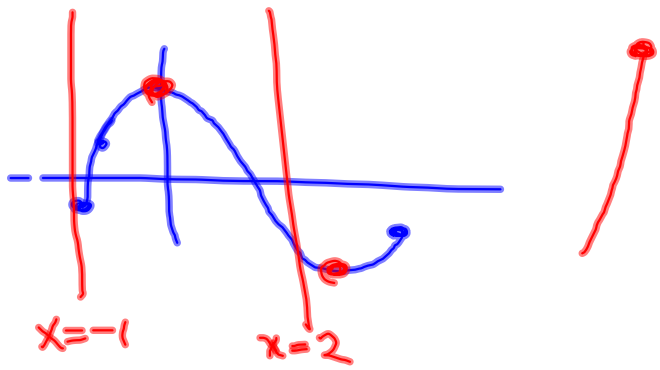
$\underline{f(0) = 2}$ $\qquad f(4):$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 0 & 2 \\ & & 4 & -8 & -32 \\ \hline & 1 & -2 & -8 & \underline{-30 = f(4)} \end{array}$$

local max @ $(0,2)$
~~local min @ $(4,-30)$~~
Abs. max @ $(0,2)$
~~Abs. min @ $(4,-30)$~~

But, Steve,
$x = 4 \notin [-1,2]$

Abs min @ $(2,-14)$



Find local and absolute extremes
for $f(x) = x^3 - 6x^2 + 2$ on $[-1, 2]$

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 0 & 2 \\ & & -1 & 7 & -7 \\ \hline & 1 & -7 & 7 & \boxed{-5 = f(-1)} \\ & x^2 & x & c & r \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 0 & 2 \\ & & 2 & -8 & -16 \\ \hline & 1 & -4 & -8 & -14 = f(2) \end{array}$$

This says $x = -1 \Longrightarrow$

$x^3 - 6x^2 + 2 = \underbrace{(x+1)}_{\to 0}(x^2 - 7x + 7) - 5$

$\dfrac{x^3 - 6x^2 + 2}{x+1} = x^2 - 7x + 7 - \dfrac{5}{x+1}$

$9 \text{ r } 1$
$3\overline{)28}$

$28 = 3 \cdot 9 + 1$

$\dfrac{28}{3} = 9 + \dfrac{1}{3}$

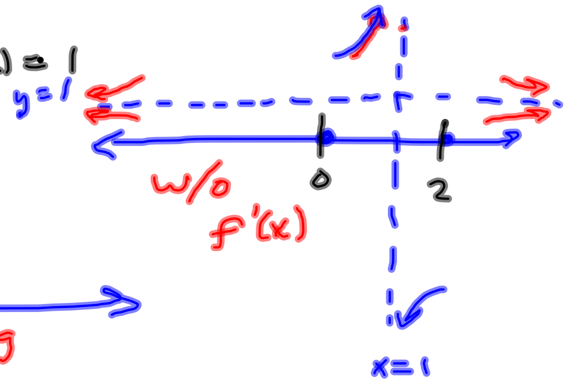
Remainder when dividing by $x - c$ is $f(c)$.

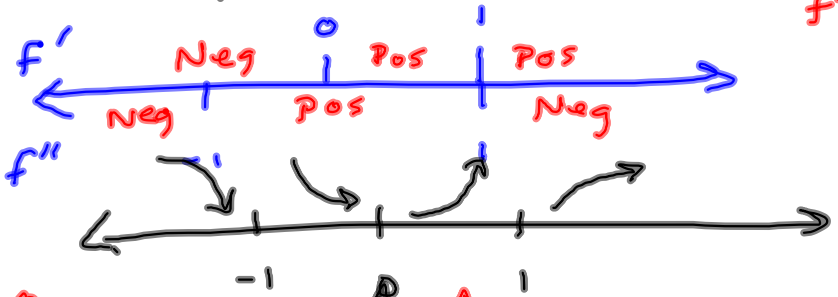
$$\lim_{x\to\infty} \frac{x^3-4x+2}{2x^3+5x-4} = \frac{1}{2}$$

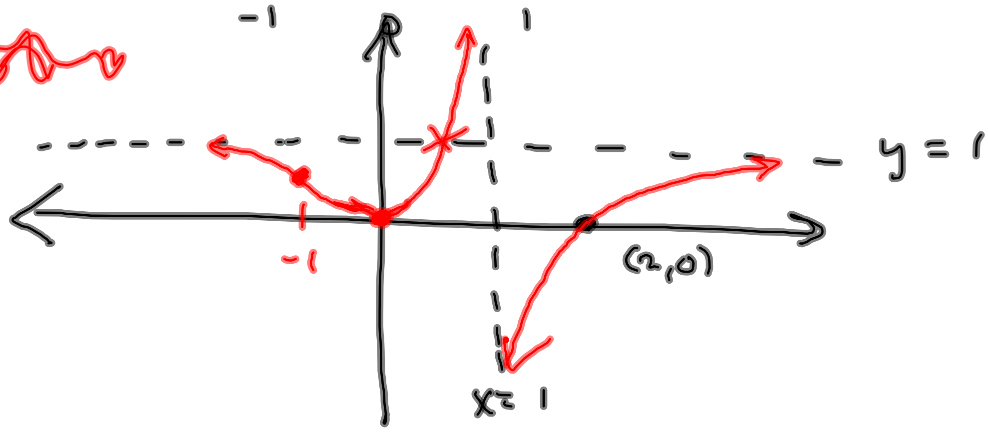
$$\lim_{x\to 1^-} f(x) = \infty, \quad \lim_{x\to 1^+} f(x) = -\infty,$$

$$\lim_{x\to -\infty} f(x) = 1, \quad \lim_{x\to\infty} f(x) = 1$$

$$f(0)=0, \quad f(2)=0$$







$f'(x) = 6x+4, \quad f(0)=3$

$f(x) = 3x^2 + 4x + C$

$f(0) = C = 3$

$f(x) = 3x^2 + 4x + 3$

$\frac{1}{2}x^4$

Apr 1-12:36 PM

Find all values c that satisfy the ~~MVT~~ Mean Value Theorem for $f(x) = x^2 - 3x + 5$ on $[-1, 3]$

$$m = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{5 - 9}{4} = \frac{-4}{4} = -1$$

$$\begin{array}{r|rrr} -1 & 1 & -3 & 5 \\ & & -1 & 4 \\ \hline & 1 & -4 & 9 = f(-1) \end{array}$$

$$\begin{array}{r|rrr} 3 & 1 & -3 & 5 \\ & & 3 & 0 \\ \hline & 1 & 0 & 5 = f(3) \end{array}$$

$$m = -1 \overset{SET}{=} f'(x)$$

$$f'(x) = 2x - 3 \overset{SET}{=} -1 \implies$$

$$2x = 2$$

$$x = \boxed{1 = c}$$

Suppose $3 \le f'(x) \le 5$ for all x, and

$\boxed{f(2)=7}$, Show that $\underline{25 \le f(8) \le 37}$

$f(2)$

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

$3 \le f'(c) \le 8$

$$3 \le \boxed{\frac{f(2)-f(8)}{2-8} = f'(c)} \le 5$$

MVT

$$\frac{7-f(8)}{-6} = f'(c) \quad \text{for some } c \in (7,8)$$

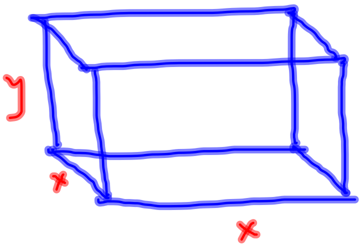
$$3 \le \frac{7-f(8)}{-6} \le 5$$

$$-18 \ge 7-f(8) \ge -30$$

$$-25 \ge -f(8) \ge -37$$

$$\underline{25 \le f(8) \le 37}$$

A metal box has a square base and no top. Its volume is 108 ft^3. What dimensions minimize the amount of metal used?



$$V = 108 = x^2 y$$

$$\Rightarrow y = \frac{108}{x^2}$$

$$S = x^2 + 4xy$$

$$= x^2 + 4x\left(\frac{108}{x^2}\right)$$

$$= x^2 + \frac{432}{x} = x^2 + 432x^{-1}$$

$$\frac{dS}{dx} = 2x - 432x^{-2}$$

$$= \frac{2x^3 - 432}{x^2} \overset{\text{SET}}{=} 0$$

2 | 212
2 | 106
53

$$\Rightarrow 2x^3 = 432$$

$$x^3 = 216$$

$$x = \sqrt[3]{216} = 6$$

$$y = \frac{108}{36} = 3$$

6 ft by 3 ft
= width by height.

$$\frac{2x^3 - 432}{x^2} = \frac{2(x^3 - 216)}{x^2}$$