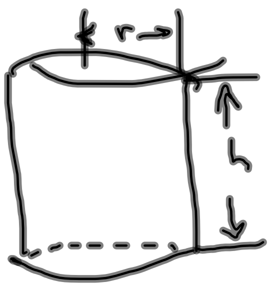



$x^2+y^2=r^2$

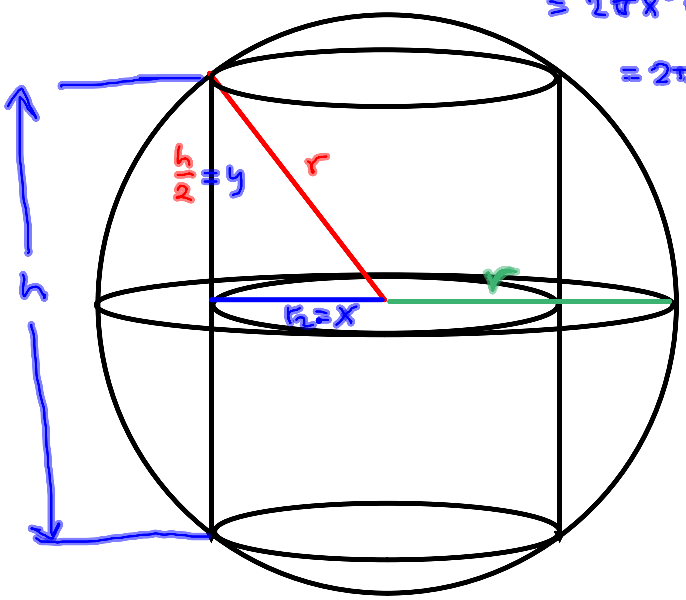
$y=\sqrt{r^2-x^2}$

Surface Area $= 2\pi r^2 + 2\pi r h$ $= 2\pi x^2 + 2\pi x \cdot 2y$

$= 2\pi x^2 + 4\pi x y$

$= 2\pi x^2 + 4\pi x\sqrt{r^2-x^2}$

$= S(x).$



$$S(x) = 2\pi x^2 + 4\pi x\sqrt{r^2-x^2} = 2\pi\left[x^2 + 2x\sqrt{r^2-x^2}\right]$$

$$S'(x) = 2\pi\left[2x + 2\sqrt{r^2-x^2} + 2x\left(\tfrac{1}{2}(r^2-x^2)^{-\frac{1}{2}}(-2x)\right)\right] \overset{SET}{=} 0$$

$$\Longrightarrow \quad 2x + 2\sqrt{r^2-x^2} - \frac{2x^2}{\sqrt{r^2-x^2}} = 0$$

$$\Longrightarrow 2x\cdot\frac{\sqrt{r^2-x^2}}{\sqrt{r^2-x^2}} + 2\sqrt{r^2-x^2}\cdot\frac{\sqrt{r^2-x^2}}{\sqrt{r^2-x^2}} - \frac{2x^2}{\sqrt{r^2-x^2}} = 0$$

$$\frac{2x\sqrt{r^2-x^2} + 2(r^2-x^2) - 2x^2}{\sqrt{r^2-x^2}} = 0$$

$x = y \Longrightarrow$

$x^2 = y^2 \Longrightarrow$

$x = \pm y \Longrightarrow$

$$x\sqrt{r^2-x^2} + r^2 - x^2 - x^2 = 0$$

Extraneous Fish.

$A = B \Longrightarrow$

$A^2 = B^2$

$$x\sqrt{r^2-x^2} + r^2 - 2x^2 = 0$$

$$x\sqrt{r^2-x^2} = 2x^2 - r^2$$

$$\left(x\sqrt{r^2-x^2}\right)^2 = \left(2x^2-r^2\right)^2$$

$(a-b)^2 =$
$a^2 - 2ab + b^2$

$$x^2(r^2-x^2) = 4x^4 - 4x^2r^2 + r^4$$

$$r^2x^2 - x^4 = 4x^4 - 4r^2x^2 + r^4$$

$$5x^4 - 5r^2x^2 + r^4 = 0$$

Let $u = x^2$, then

$$5u^2 - 5r^2u + r^4 = 0$$

$a = 5,\ b = -5r^2,\ c = r^4$

$$b^2 - 4ac = (5r^2)^2 - 4(5)(r^4) = 25r^4 - 20r^4$$
$$= 5r^4$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{5r^2 \pm \sqrt{5r^4}}{10} = u = x^2$$

$$= \frac{5r^2 \pm \sqrt{5}\,r^2}{10} = \frac{5\pm\sqrt{5}}{10}r^2 = x^2$$

$$\Rightarrow \boxed{x = \pm\sqrt{\frac{5\pm\sqrt{5}}{10}}\,r}$$

Test for sign of $f'(x)$

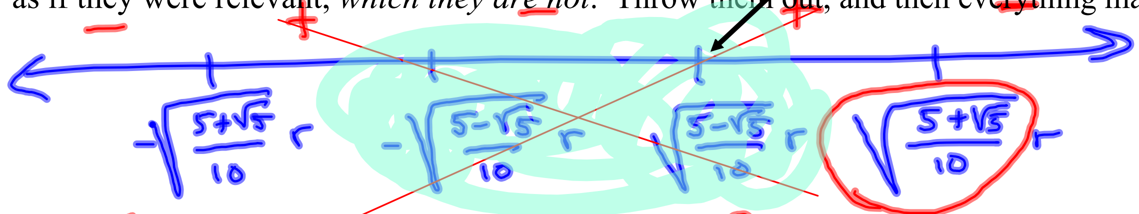
We're good up to this point. But I let you guys down on the subsequent analysis. The 5 - sqrt(5) version doesn't satisfy the following equation:

$\sqrt{\frac{5-\sqrt{5}}{10}}\,r$ doesn't work.

$$x\sqrt{r^2-x^2} = 2x^2 - r^2$$

Bad stuff.

... because the left-hand side is positive and the right-hand side is negative. So then it becomes matter of analyzing the sign to either side of the right-most root of DA/Dx, to make sure it gives max. I'm still not happy with the sign pattern, below, because it includes the 5 - sqrt(5) thingies as if they were relevant, *which they are not*. Throw them out, and then everything makes sense.




Do sign pattern for next time

$\sqrt{\frac{5-\sqrt{5}}{10}}\,r$ doesn't work.

$$x\sqrt{r^2-x^2} = 2x^2-r^2$$

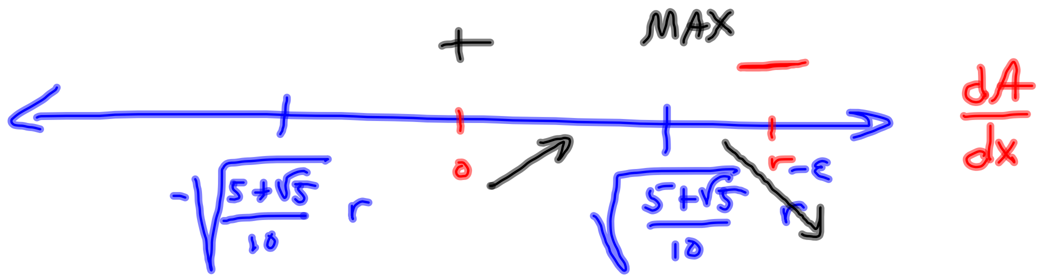
$$\underbrace{\sqrt{\frac{5-\sqrt{5}}{10}}\,r\sqrt{r^2-\frac{5-\sqrt{5}}{10}r^2}}_{\text{positive}} \overset{?}{=} \cancel{2}\,\frac{5-\sqrt{5}}{\cancel{10}}r^2 - r^2$$

$$= \frac{5-\sqrt{5}}{5}r^2 - r^2$$

$$= 1r^2 - \frac{\sqrt{5}}{5}r^2 - r^2$$

$$= \underbrace{-\frac{\sqrt{5}}{5}r^2}_{\text{negative}}$$

Nope. This means $\sqrt{\frac{5-\sqrt{5}}{10}}\,r$ is extraneous. Throw that fish back.




$$\left.\frac{dA}{dx}\right|_{x=0} = 4\pi \left[ \frac{x\sqrt{r^2-x^2} + r^2 - 2x^2}{\sqrt{r^2-x^2}} \right]\Bigg|_{x=0} = 4\pi \left[ \frac{r^2}{\sqrt{r^2}} \right]$$

$$\left.\frac{dA}{dx}\right|_{x=r-\varepsilon} = 4\pi \left[ \frac{\underbrace{(r-\varepsilon)\sqrt{r^2-(r-\varepsilon)^2}}_{\to 0} + r^2 \underbrace{-2(r-\varepsilon)^2}_{\to -2r^2}}{\underbrace{\sqrt{r^2-(r-\varepsilon)^2}}_{\to 0 \text{ but not quite} = 0}} \right]$$

I can't quite get away with this argument.

And after a miracle, we conclude $\frac{dA}{dx} < 0$ to the right of $x = \sqrt{\frac{5+\sqrt{5}}{10}}\, r$

Trying a specific value of $r$, for instance $r=1$ & THEN looking at this, would give you the sign pattern and conclude that $\sqrt{\frac{5+\sqrt{5}}{10}}\, r$ corresponds to a max.

#31

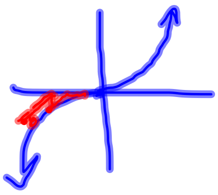
$$f(x) = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

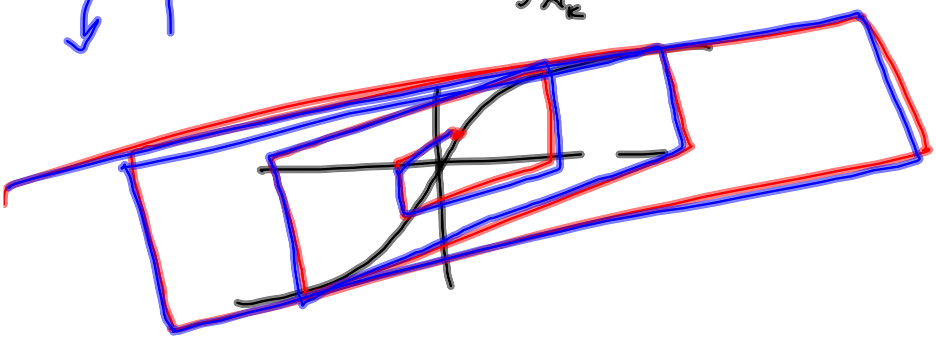
$x^{\frac{m}{n}}$

$\sqrt[3]{-8} = -2$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$



$$= x_k - \frac{x_k^{\frac{1}{3}}}{\frac{1}{3}x_k^{-2/3}} = x_k - 3x_k^{\frac{1}{3}+\frac{2}{3}} = x_k - 3x_k = -2x_k$$



## Some 4.8 stuff:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$Y_1 : f(x)$

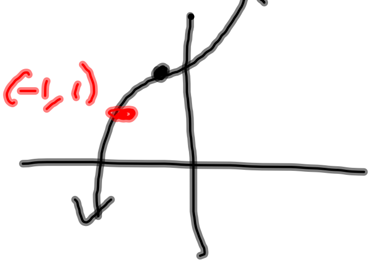
$Y_2 : f'(x)$

$Y_3 : x - Y_1 / Y_2$

#9 $f(x) = x^3 + x + 3$

$f'(x) = 3x^2 + 1$

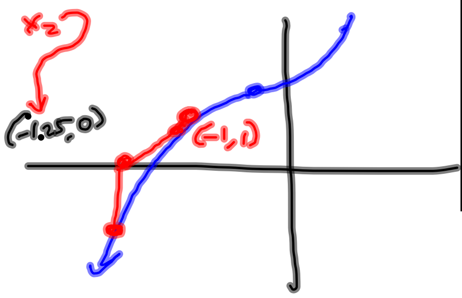
Quick Graph




$x_1 = -1$

```
 Plot1 Plot2 Plot3
\Y1=X^3+X+1+2
\Y2=3X²+1
\Y3=X-Y1/Y2
\Y4=
\Y5=
\Y6=
\Y7=
```

Newton's on graphing calculator




```
Y3(-1)
                -1.25
Y3(Ans)
         -1.214285714
         -1.213412176
         -1.213411663
         -1.213411663
```

§4.9 #66

$50 = v_0$

$a = -22 \frac{ft}{s^2}$

Find distance b4 car stops.

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$ is one way.

$\frac{1}{2}(-22)t^2 + 50t + 0$

$-11t^2 + 50t$

$a = -22 \implies$

$v = -22t + C$

$v(0) = -22(0) + C = 50$

$\implies C = 50 = v_0$

$v(t) = -22t + 50$

$s(t) = \frac{-22}{2}t^2 + 50t + d$

Let $s(0) = 0$. Then $d = 0$

$s(t) = -11t^2 + 50t$

$v(t) = 0$ STOP

$t = \frac{50}{22} = \frac{25}{11}$

$s\left(\frac{25}{11}\right)$ is the answer.

4.7 5pm 2day
4.8 Tomorrow
4.9 I Tomorrow
4.9 II Friday.