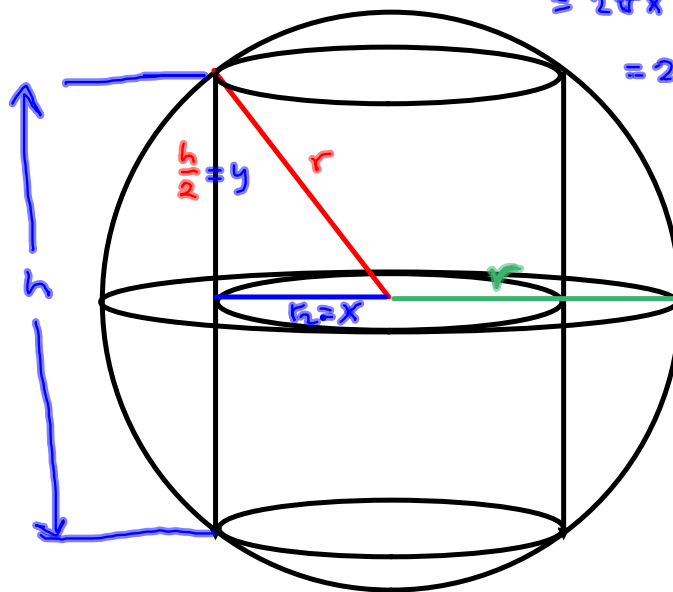


$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2}$$

$$\begin{aligned} \text{Surface Area} &= 2\pi r^2 + 2\pi r h = 2\pi x^2 + 2\pi x \cdot 2y \\ &= 2\pi x^2 + 4\pi x y \\ &= 2\pi x^2 + 4\pi x \sqrt{r^2 - x^2} \\ &= S(x). \end{aligned}$$



$$S(x) = 2\pi x^2 + 4\pi x \sqrt{r^2 - x^2} = 2\pi \left[ x^2 + 2x \sqrt{r^2 - x^2} \right]$$

$$S'(x) = 2\pi \left[ 2x + 2\sqrt{r^2 - x^2} + 2x \left( \frac{1}{2} (r^2 - x^2)^{-\frac{1}{2}} (-2x) \right) \right] \stackrel{SET}{=} 0$$

$$\Rightarrow 2x + 2\sqrt{r^2 - x^2} - \frac{2x^2}{\sqrt{r^2 - x^2}} = 0$$

$$\Rightarrow 2x \cdot \frac{\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}} + 2\sqrt{r^2 - x^2} \cdot \frac{\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}} - \frac{2x^2}{\sqrt{r^2 - x^2}} = 0$$

$$\frac{2x\sqrt{r^2 - x^2} + 2(r^2 - x^2) - 2x^2}{\sqrt{r^2 - x^2}} = 0$$

$$\Rightarrow x\sqrt{r^2 - x^2} + r^2 - x^2 - x^2 = 0$$

$$x\sqrt{r^2 - x^2} + r^2 - 2x^2 = 0$$

$$x\sqrt{r^2 - x^2} = 2x^2 - r^2$$

$$\left( x\sqrt{r^2 - x^2} \right)^2 = \left( 2x^2 - r^2 \right)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x^2(r^2 - x^2) = 4x^4 - 4x^2r^2 + r^4$$

$$\overbrace{r^2x^2 - x^4} = \overbrace{4x^4 - 4r^2x^2 + r^4}$$

$$5x^4 - 5r^2x^2 + r^4 = 0$$

Let  $u = x^2$ , then

$$5u^2 - 5r^2u + r^4 = 0$$

$$a=5, b=-5r^2, c=r^4$$

$$b^2 - 4ac = (5r^2)^2 - 4(5)(r^4) = 25r^4 - 20r^4 = 5r^4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5r^2 \pm \sqrt{5r^4}}{10} = u = x^2$$

$$= \frac{5r^2 \pm \sqrt{5}r^2}{10} = \frac{5 \pm \sqrt{5}}{10} r^2 = x^2$$

$$\Rightarrow x = \pm \sqrt{\frac{5 \pm \sqrt{5}}{10}} r$$

Test for sign of  $f'(x)$

We're good up to this point. But I let you guys down on the subsequent analysis. The 5 - sqrt(5) version doesn't satisfy the following equation:

$$x\sqrt{r^2 - x^2} = 2x^2 - r^2$$

Bad stuff.

... because the left-hand side is positive and the right-hand side is negative. So then it becomes a matter of analyzing the sign to either side of the right-most root of  $DA/Dx$ , to make sure it gives a max. I'm still not happy with the sign pattern, below, because it includes the 5 - sqrt(5) thingies, as if they were relevant, which they are not. Throw them out, and then everything makes sense.

