4.9 - Antiderivatives

4.8	#s 5, 8, 9, 16, 17, 22, 29, 31
4.9	#s 1, 6, 13, 20, 21, 26, 31, 40, 44, 45, 46
4.9 II	#s 51, 58, 60, 66, 67

DEFINITION A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

It says "*an* antiderivative." How many antiderivatives are there for a given function f?

an infinite # (uncountably so).

$f(x) = x-3$

$F(x) = \frac{x^2}{2} - 3x + C$, where C is any real number.

1, 2, 3, 4, ... countably many.

Check: $F'(x) = 2\left(\frac{x}{2}\right) - 3 = x-3$ ✓

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$$\frac{d}{dx}[\sin x] = \cos x$$

$$\int \cos x \, dx = \sin x + C$$

Indefinite integral ≈ Antiderivative

$$\frac{d}{dx}[\sin x + C] = \cos x \checkmark$$

$$\int \sec^2 x \, dx = \tan x + C$$

Table of Trig derivatives in reverse.

Power Rule for Antiderivatives:

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

$$f(x) = 3x^6 \Longrightarrow F(x) = \frac{3}{7} x^7$$

Find $f(x)$ if $\boxed{f''(x) = 6x + 12x^2} = 12x^2 + 6x$

$f'(x) = 6 \cdot \frac{1}{2}x^2 + 12 \cdot \frac{1}{3}x^3 \boxed{+C}$

$= \boxed{3x^2 + 4x^3 + C}$

Check: $\frac{d}{dx}[\nearrow] = 6x + 12x^2$ ✓

Beginning Differential Equations.

$f(x) = 3 \cdot \frac{1}{3}x^3 + 4 \cdot \frac{1}{4}x^4 + Cx \boxed{+d}$

$= \boxed{x^3 + x^4 + Cx + d.}$

Constants 'c' & 'd' depend on "where it started."

Finding f from f'' & "initial conditions" on f', f.

where it started.

35 $f''(\theta) = \sin(\theta) + \cos\theta \Longrightarrow$

$\boxed{\begin{array}{l} f'(0)=4 \\ f(0)=3 \end{array}}$ Given also.

$f'(\theta) = -\cos(\theta) + \sin(\theta) + C$

$f'(0) = -\cos(0) + \sin(0) + C = 4$

$= -1 + 0 + C = 4$

$\Longrightarrow C = 5$

$\therefore f'(\theta) = -\cos(\theta) + \sin(\theta) + 5$

$f(\theta) = -\sin(\theta) - \cos(\theta) + 5\theta + d$

$f(0) = -\sin(0) - \cos(0) + (5)(0) + d = 3$

$= -1 + d = 3$

$d = 4$

$$\therefore \boxed{f(\theta) = -\sin(\theta) - \cos(\theta) + 5\theta + 4}$$

Check by $\frac{d}{d\theta}[f(\theta)]$

Boundary Conditions

39 $f''(x) = 2 + \cos(x)$, $f(0) = -1$, $f(\frac{\pi}{2}) = 0$

$f'(x) = 2x + \sin(x) + C$

$f(x) = x^2 - \cos(x) + Cx + d$

$f(0) = 0^2 - \cos(0) + (C)(0) + d = -1$

$-1 + d = -1$

$d = 0 \Longrightarrow f(x) = x^2 - 2\cos(x) + Cx$

$f(\frac{\pi}{2}) = (\frac{\pi}{2})^2 - \cos(\frac{\pi}{2}) + C(\frac{\pi}{2}) = 0$

$\frac{\pi^2}{4} + \frac{\pi}{2}C = 0$

$\frac{\pi}{2}C = -\frac{\pi^2}{4}$

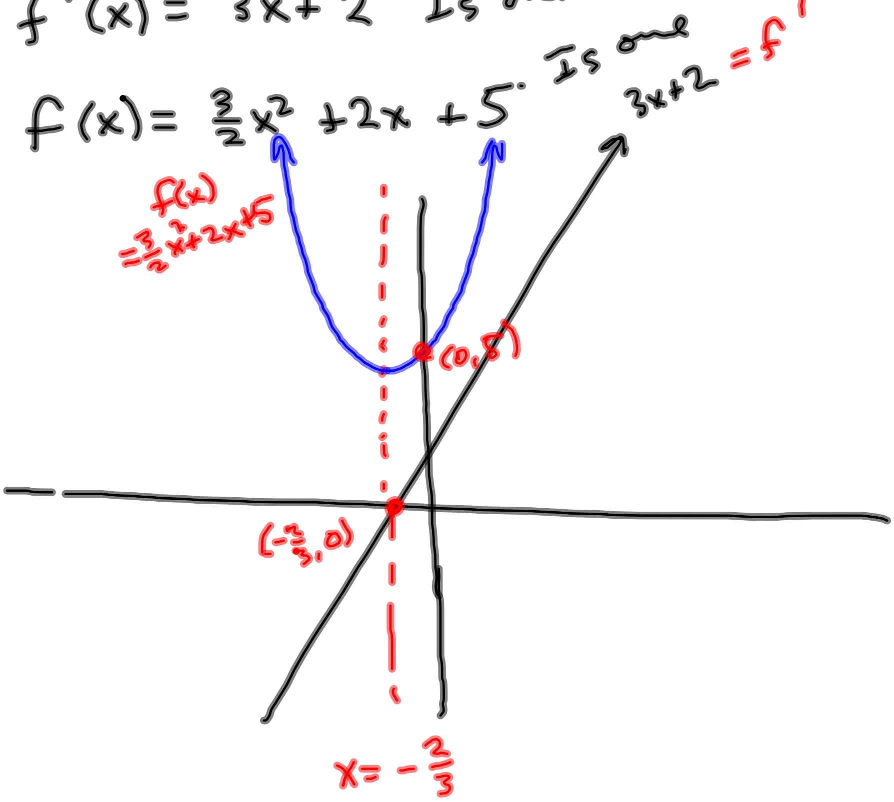
$C = -\frac{\pi^2}{4} \cdot \frac{2}{\pi} = -\frac{\pi}{2}$

$\boxed{f(x) = x^2 - \cos(x) - \frac{\pi}{2}x}$

$f''(x) = 3$

$f'(x) = 3x + 2$ Is one

$f(x) = \frac{3}{2}x^2 + 2x + 5$. Is one



$f''(x) = 2x+1$ $\qquad$ $f(0)=1$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $f(1)=5$

$f'(x) = x^2 + x + C$

$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + cx + d$

$f(0) = d = 1$

$f(1) = \frac{1}{3} + \frac{1}{2} + c + 1 = 5$

$\frac{2+3+6}{6} + c = 5$

$\frac{11}{6} + c = 5$

$c = 5 - \frac{11}{6} = \frac{30-11}{6} = \frac{19}{6}$

$\therefore\ f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{19}{6}x + 1$

Revenue = Price * # sold.

Cost =

Profit = Revenue − Cost

Price is the variable, x, in dollars.

~~(x, R) = (10, 20) means R(10) = 20~~

~~Revenue = R = R(x)~~

~~We were also given R(11) = 18~~

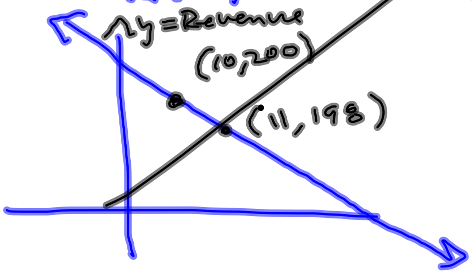
No, Steve. 20 is the # sold per day @ x = $10/rock

$R(10) = 20 \cdot 10 = 200$

$(10, 200)$

$R(11) = 18 \cdot 11 = 198 \rightsquigarrow (11, 198)$

18
180

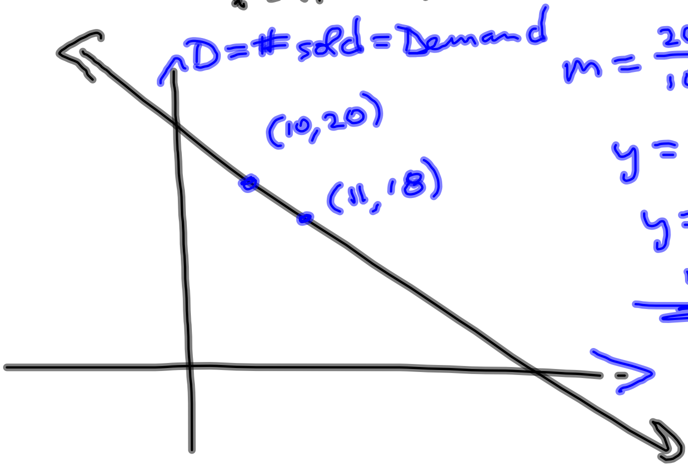


You still don't get it.

Assume Demand is a linear function

Then $x = 10 \rightsquigarrow D = 20$

$x = 11 \rightsquigarrow D = 18$



$m = \frac{20-18}{10-11} = \frac{-2}{1} = -2$

$y = m(x - x_1) + y_1$

$y = -2(x-10) + 20$

$\underline{y = -2x + 40}$ = Demand = $D(x)$

Maximize Profit

$= R(x) - C(x)$

$= D(x) \cdot x - D(x) \cdot 6$

$= (-2x+40)x - (-2x+40)(6)$

$C(x) = 6 \cdot D(x)$

(cost per item) times (# of items made)

if he makes as many as he sells.