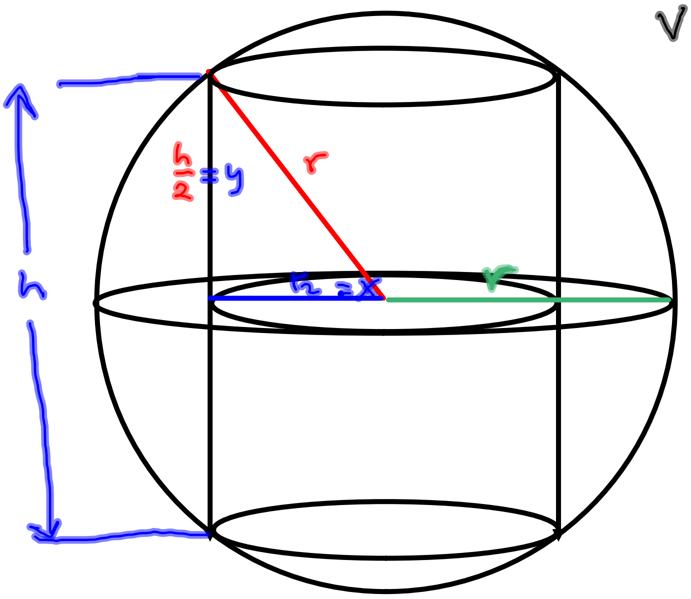
§4.7 #27



$V =$ Volume

$= \pi r_2^2 h$

$\left(\frac{h}{2}\right)^2 + r_2^2 = r^2$

My way

$y^2 + x^2 = r^2$

Book Way

$V = \pi r_2^2 \cdot 2\sqrt{r^2 - r_2^2} = 2\pi r_2^2 \sqrt{r^2 - r_2^2}$

$\frac{dV}{dr_2} = 2\pi \left[2r_2 \sqrt{r^2 - r_2^2} + r_2^2 \left(\frac{1}{2} (r^2 - r_2^2)^{-\frac{1}{2}} (-2r_2) \right) \right]$

If you're stupid & do it my way.

Maximize Volume.

V = Volume

$= \pi r_2^2 h$

$\left(\frac{h}{2}\right)^2 + r_2^2 = r^2 \Longrightarrow \frac{h^2}{4} = r^2 - r_2^2$

My way

$h^2 = 4(r^2 - r_2^2)$

$h = |h| = \sqrt{4(r^2 - r_2^2)}$, since $h > 0$.

$= 2\sqrt{r^2 - r_2^2}$

r_2 might be awkward.

Let $r_2 = x$, for ease and comfort.

$V = \pi r_2^2 h = \pi x^2 h = \pi x^2 \left(2\sqrt{r^2 - x^2}\right)$

$= 2\pi x^2 \sqrt{r^2 - x^2} = \underline{2\pi x^2 (r^2 - x^2)^{\frac{1}{2}}}$

$$\frac{dV}{dx} = 2\pi\left[2x(r^2 - x^2)^{\frac{1}{2}} + x^2\left(\frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}(-2x)\right)\right]$$

$$= 2\pi\left[2x\sqrt{r^2 - x^2} - \frac{x^3}{\sqrt{r^2 - x^2}}\right]$$

$$= 2\pi\left[2x\sqrt{r^2 - x^2} \cdot \frac{\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}} - \frac{x^3}{\sqrt{r^2 - x^2}}\right]$$

$$= 2\pi\left[\frac{2x(r^2 - x^2) - x^3}{\sqrt{r^2 - x^2}}\right] \overset{\text{SET}}{=} 0$$

(Critical #s: where $f'(x) = 0$ or $f'(x) \nexists$ but $x \in D(f)$)

$r = x$... Boring.

So we want $2x(r^2 - x^2) - x^3 = 0$

$x(2(r^2 - x^2) - x^2) = 0$

$x(2r^2 - 2x^2 - x^2) = 0$

$x = 0$ ... Boring

$2r^2 - 3x^2 = 0$

$3x^2 = 2r^2$

$x^2 = \frac{2}{3} r^2$

$x = |x| = \sqrt{\frac{2}{3}}\, r = \frac{\sqrt{6}\, r}{3}$

And so, the volume is maximized when $x = \frac{\sqrt{6}}{3} r$ & the volume is

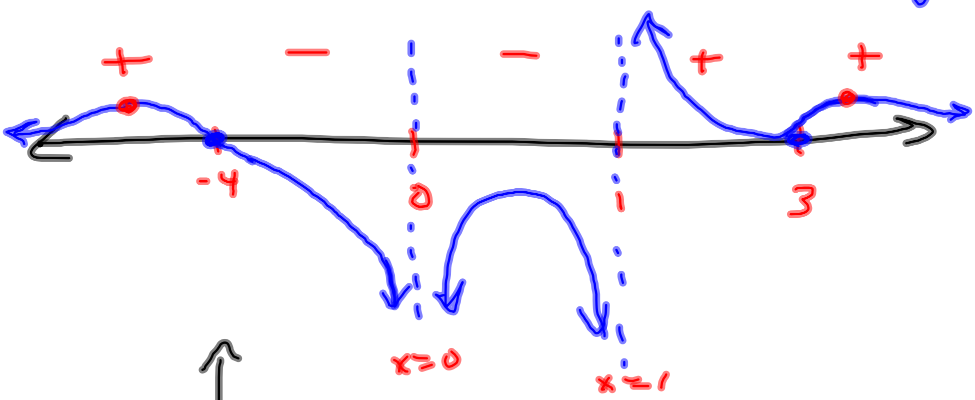
$$V = 2\pi x^2 \sqrt{r^2 - x^2}\Big|_{x = \frac{\sqrt{6}}{3} r} = 2\pi \left(\frac{\sqrt{6}}{3} r\right)^2 \sqrt{r^2 - \left(\frac{\sqrt{6}}{3} r\right)^2}$$

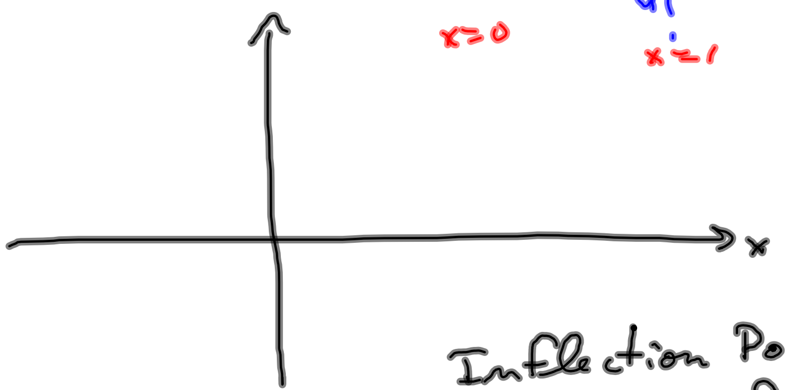
$$2 \cdot \frac{2\pi r^2}{3} \sqrt{r^2 - \frac{2}{3} r^2} = \frac{2\pi r^2}{3} \sqrt{\frac{1}{3} r^2} \cdot 2$$

$$= \frac{2\pi r^2}{3} \sqrt{\frac{1}{3}}\, r \cdot 2 = 2 \cdot \frac{2\pi r^3}{3\sqrt{3}} = \boxed{\frac{4\pi\sqrt{3}}{9} r^3}$$

$\boxed{4.6 \ \#11}$

$$\frac{(x+4)(x-3)^2}{x^4(x-1)} = \frac{\deg 3}{\deg 5} \Rightarrow \text{Proper}$$
$$\Rightarrow y=0 \text{ is H.A.}$$





Inflection Points not necessary for #12, but use a grapher to find max/min. values.