43–44

(a) Use a graph of f to estimate the maximum and minimum values. Then find the exact values.

(b) Estimate the value of x at which f increases most rapidly. Then find the exact value.

43. $f(x) = \dfrac{x+1}{\sqrt{x^2+1}}$

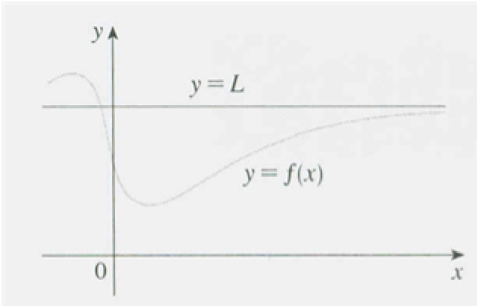
http://people.hofstra.edu/stefan_waner/realworld/functions/func.html

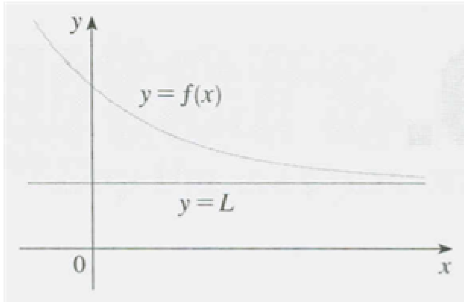
4.4 LIMITS AT INFINITY; HORIZONTAL ASYMPTOTES

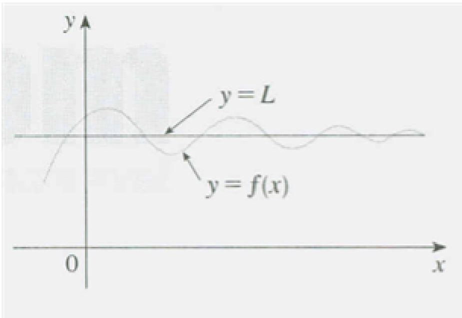
1 DEFINITION Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

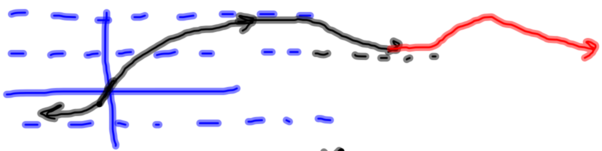
means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.







How many H.A. can a function have? 2



$\frac{x}{\sqrt{x^2+1}}$

2 DEFINITION Let f be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x \to -\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large negative.

3 DEFINITION The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L_1 \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L_2$$

$L_1 = 1$, $L_2 = -1$

Main examples of horizontal asymptotes: Rational functions where the degree of the numerator is the same as the degree of the denominator.

If degree of denominator is Greater than the degree of the numerator, then $R(x) = \frac{p(x)}{q(x)}$ is PROPER and $y = 0$ is the HA.

9–30 Find the limit.

10. $\lim_{x\to\infty} \frac{3x+5}{x-4}$ $= 3$

$\lim_{|x|\to\infty} \frac{a}{x} = 0$

$$\frac{3x+5}{x-4} = \frac{x\left(3+\frac{5}{x}\right)}{x\left(1-\frac{4}{x}\right)} = \frac{3+\frac{5}{x}}{1-\frac{4}{x}} \xrightarrow{x\to\infty} \frac{3}{1} = 3$$

$\frac{5}{x} \to 0$, $\frac{4}{x} \to 0$

13. $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$

17. $\lim_{x\to\infty} \frac{\sqrt{9x^6 - x}}{x^3+1}$ $= \lim_{x\to\infty} \frac{\sqrt{9x^6(1-\frac{1}{9x^5})}}{x^3(1+\frac{1}{x^3})}$. Now,

$$\frac{\sqrt{9x^6-x}}{x^3+1} = \frac{3|x|^3\sqrt{1-\frac{1}{9x^5}}}{x^3(1+\frac{1}{x^3})} = \frac{3x^3\sqrt{1-\frac{1}{9x^5}}}{x^3(1+\frac{1}{x^3})}$$

(since $x\to\infty$ implies, in particular, that $x>0$)

$$= \frac{3\sqrt{1-\frac{1}{9x^5}}}{1+\frac{1}{x^3}} \xrightarrow{x\to\infty} 3$$

The following is an "odd" application of the "rationalization technique."

$$(3x)^2 = 3^2x^2 = 9x^2$$

$\boxed{19.}$ $\lim_{x\to\infty}(\sqrt{9x^2+x} - 3x)$ $= \infty - \infty$!?

$$= \lim_{x\to\infty}\left[\left(\frac{\sqrt{9x^2+x} - 3x}{1}\right)\left(\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right)\right]$$

$$= \lim_{x\to\infty}\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty}\frac{x}{\sqrt{9x^2(1+\frac{1}{9x})}+3x} = \lim_{x\to\infty}\frac{x}{|3x|\sqrt{1+\frac{1}{9x}}+3x}$$

$$= \lim_{x\to\infty}\frac{x}{3x\sqrt{1+\frac{1}{9x}}+3x}$$

$$= \lim_{x\to\infty}\frac{\cancel{x}}{3\cancel{x}\left(\sqrt{1+\frac{1}{9x}}+1\right)} = \lim_{x\to\infty}\frac{1}{3\left(\sqrt{1+\frac{1}{9x}}+1\right)}$$

$$= \frac{1}{3(\sqrt{1}+1)} = \frac{1}{3(1+1)} = \frac{1}{3(2)} = \frac{1}{6}$$

So, in a sense, $\sqrt{9x^2+x} \approx 3x+\frac{1}{6}$ as $x\to\infty$!?

$$\lim_{x\to\infty}(\cos x) \;\cancel{\exists}$$

$\boxed{4}$ **THEOREM** If $r > 0$ is a rational number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

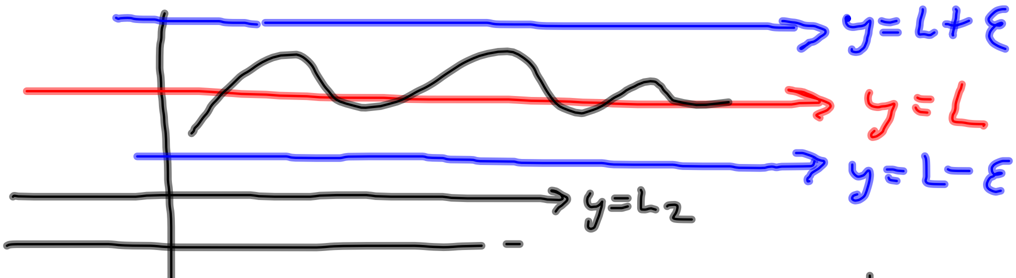
If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

> 5 **DEFINITION** Let f be a function defined on some interval (a, ∞). Then
>
> $$\lim_{x \to \infty} f(x) = L$$
>
> means that for every $\varepsilon > 0$ there is a corresponding number N such that
>
> $$\text{if} \quad x > N \quad \text{then} \quad |f(x) - L| < \varepsilon$$

We have a small number of exercises based on this formal definition, but we won't approach them in general; rather, we will be given a fixed epsilon, and then use a grapher to find an appropriate value of N.

PONDER



$X = N$

This is why you can't have more than one H.A.

$x \to \infty$ for $f(x)$ off to the right

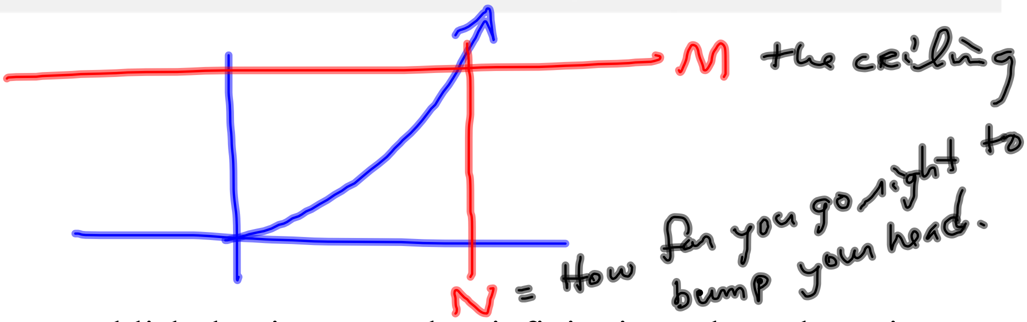
$x \to -\infty$ And no more than one off to the left

7 **DEFINITION** Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = \infty$$

means that for every positive number M there is a corresponding positive number N such that

$$\text{if} \quad x > N \qquad \text{then} \qquad f(x) > M$$



Basically, to establish that it approaches infinity is to show that, given any big number M, you can find a value of $x = N$ such that the function (y) stays above M whenever $x > N$.

"I can make it bigger than any fixed number. That means it's approaching infinity."

The closest thing in practice
to $\lim_{x\to\infty} f(x) = \infty$ is End Behavior:

$$f(x) = 11x^4 - 3x^3 + 4x^2 - 571$$

What power function does $f(x)$ resemble
as $|x| \longrightarrow \infty$?

$11x^4$

$-13x^5$ + smaller degree terms.

See #48.

§4.4 Get Crackin'
Due Friday.

4.3 # 39

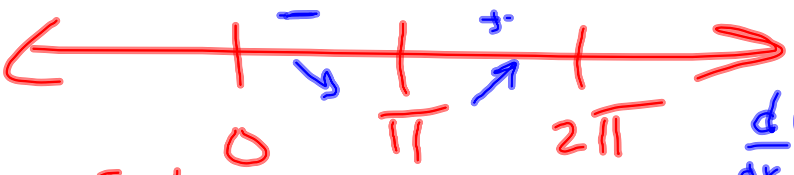
$f'(\theta) = -2\sin\theta(1 + \cos\theta) \overset{SET}{=} 0$

$f'(\theta) = 0$ @

$\Rightarrow -2\sin\theta = 0$ OR $1 + \cos\theta = 0$

$\sin\theta = 0$ OR $\cos\theta = -1$

$0, \pi, 2\pi$ $\qquad \pi$



Test

$(0,\pi)$	$\frac{\pi}{2}$	$f'(\frac{\pi}{2}) = -2 < 0$
$(\pi,2\pi)$	$\frac{3\pi}{2}$	$f'(\frac{3\pi}{2}) = +2 > 0$

$\frac{d}{dx}\left[(x^2+2)^5\right] =$

$5(x^2+2)^4(2x)$

$= \frac{d}{dx}\left[(\cos x)^2\right]$

$= 2(\cos x)^1(-\sin x)$

$2f(x)f'(x)$