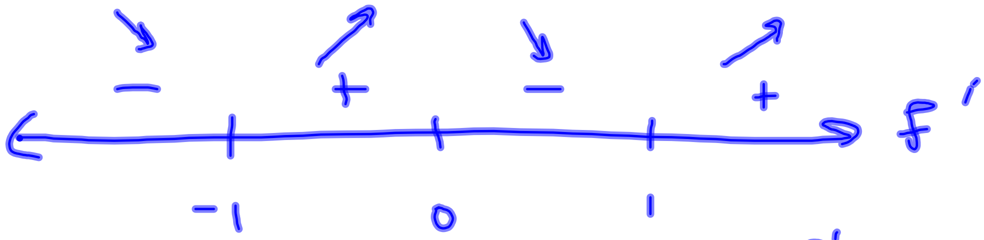
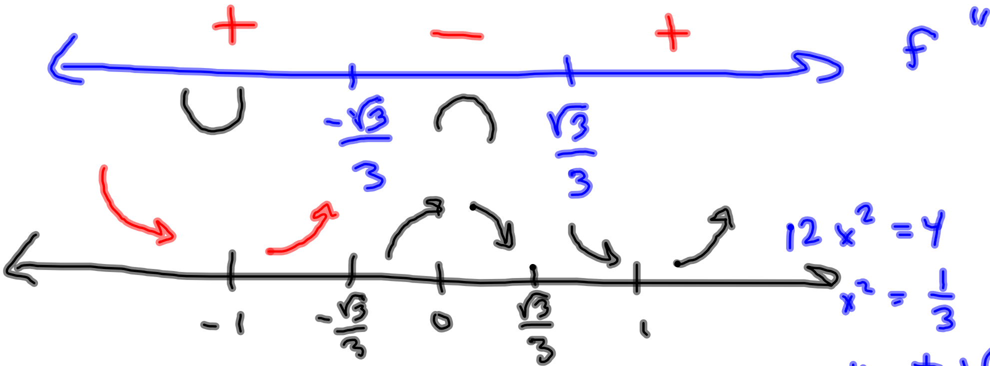


Some stuff for 4.3 #11.

$f'(2)$



$12x^2 = 4$

$x^2 = \frac{1}{3}$

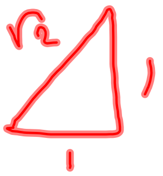
$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

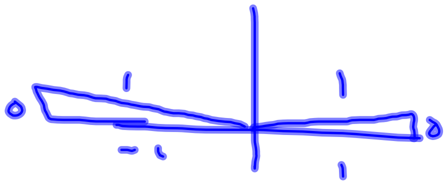
59. Show that $\tan x > x$ for $0 < x < \pi/2$. [*Hint:* Show that $f(x) = \tan x - x$ is increasing on $(0, \pi/2)$.]

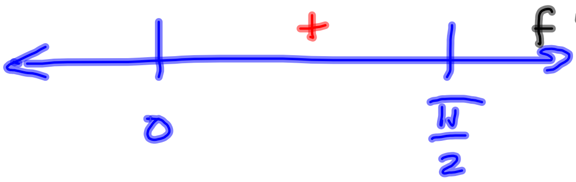
$f'(x) = \sec^2 x - 1 \overset{\text{SET}}{=} 0$

$\sec^2 x = 1$

$x = 0,\ x = \pi$







f' $\sec^2 x - 1$

Test $x = \frac{\pi}{4}$

$\sec^2\left(\frac{\pi}{4}\right) - 1 =$

$2 - 1 = 1$ +

$f'(x) > 0$ on $\left(0, \frac{\pi}{2}\right)$

$\tan x - x$ is increasing on $\left(0, \frac{\pi}{2}\right)$

$\tan(0) - 0 = 0$

$\tan(x) - x > 0$ on $\left(0, \frac{\pi}{2}\right)$

$\tan(x) > x$

43–44

(a) Use a graph of f to estimate the maximum and minimum values. Then find the exact values.

(b) Estimate the value of x at which f increases most rapidly. Then find the exact value.

43. $f(x) = \dfrac{x + 1}{\sqrt{x^2 + 1}}$

http://people.hofstra.edu/stefan_waner/realworld/functions/func.html

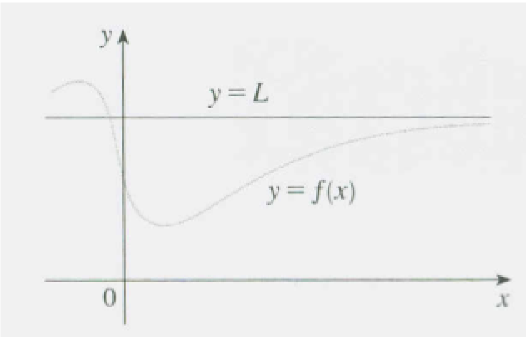
Mar 2-10:48 AM

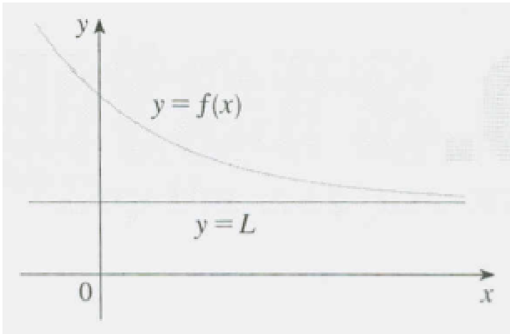
4.4 LIMITS AT INFINITY; HORIZONTAL ASYMPTOTES

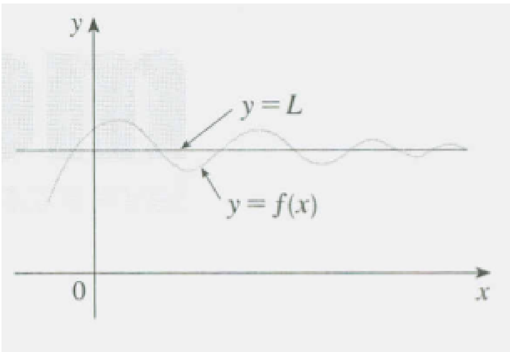
1 **DEFINITION** Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.







2 **DEFINITION** Let f be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x \to -\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large negative.

3 **DEFINITION** The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L$$

Main examples of horizontal asymptotes: Rational functions where the degree of the numerator is the same as the degree of the denominator.

Mar 6-10:02 AM

9–30 Find the limit.

10. $\lim_{x\to\infty} \frac{3x+5}{x-4}$

13. $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$

17. $\lim_{x \to \infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

The following is an "odd" application of the "rationalization technique."

19. $\lim_{x \to \infty} \left(\sqrt{9x^2 + x} - 3x\right)$

4 **THEOREM** If $r > 0$ is a rational number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

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5 DEFINITION Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that for every $\varepsilon > 0$ there is a corresponding number N such that

$$\text{if} \quad x > N \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$

We have a small number of exercises based on this formal definition, but we won't approach them in general; rather, we will be given a fixed epsilon, and then use a grapher to find an appropriate value of N.

7 **DEFINITION** Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = \infty$$

means that for every positive number M there is a corresponding positive number N such that

$$\text{if} \quad x > N \qquad \text{then} \qquad f(x) > M$$

Basically, to establish that it approaches infinity is to show that, given any big number M, you can find a value of $x = N$ such that the function (y) stays above M whenever $x > N$.

"I can make it bigger than any fixed number. That means it's approaching infinity."

Mar 6-10:11 AM