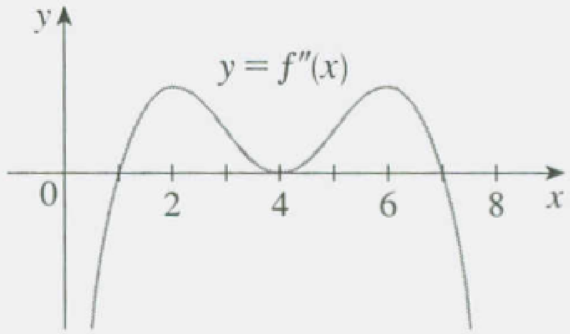
7. The graph of the second derivative f'' of a function f is shown. State the x-coordinates of the inflection points of f. Give reasons for your answers.



#14

$f := x \rightarrow \cos(x)^2 - 2 \cdot \sin(x)$

$$x \rightarrow \cos(x)^2 - 2\sin(x) = f(x)$$

$fp := D(f)$

$$x \rightarrow -2\cos(x)\sin(x) - 2\cos(x) = f'(x)$$

$fpp := D(fp)$

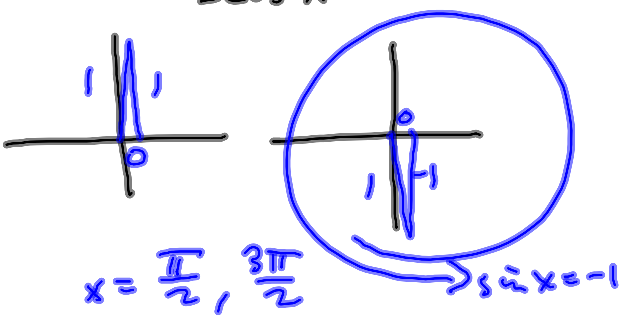
$$x \rightarrow 2\sin(x)^2 - 2\cos(x)^2 + 2\sin(x) = f''(x)$$

$f'(x) = 0 \Rightarrow -2\cos x(\sin x + 1) = 0$

$\Rightarrow -2\cos x = 0$ OR $\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

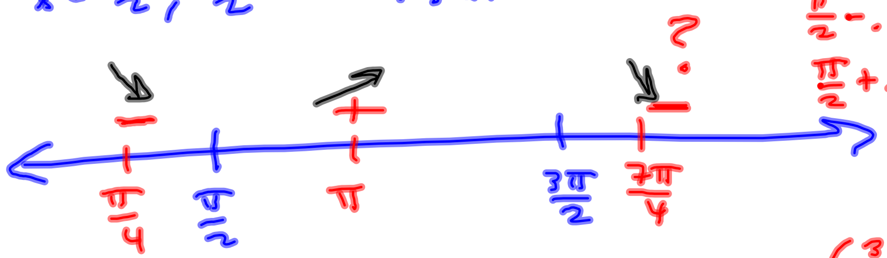


$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Test:

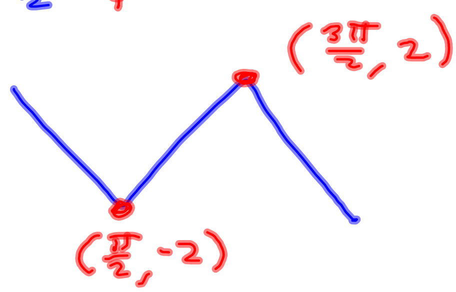
$\frac{\pi}{2} - .01$

$\frac{\pi}{2} + .01$



$f(\frac{\pi}{2}) = -2$

$f(\frac{3\pi}{2}) = 2$



$\sin^2 x + \cos^2 x = 1$

$f''(x) = 2\sin^2 x - 2\cos^2 x + 2\sin x \overset{\text{SET}}{=} 0$

$\Rightarrow 2\sin^2 x - 2(1 - \sin^2 x) + 2\sin x$

$= 2\sin^2 x - 2 + 2\sin^2 x + 2\sin x$

$= 4\sin^2 x + 2\sin x - 2 = 0$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

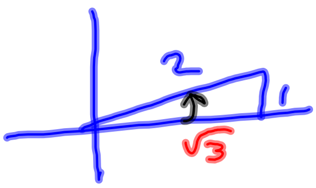
Let $u = \sin x$

$2u^2 + u - 1 = 0$

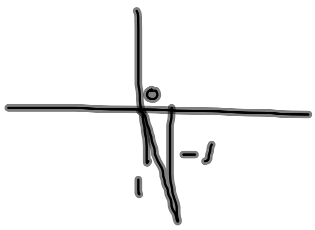
$(2u - 1)(u + 1) = 0$

$u = \frac{1}{2} \qquad u = -1$

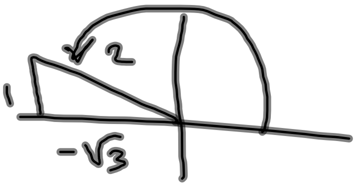
$\sin x = \frac{1}{2}$ OR $\sin x = -1$



$\frac{\pi}{6}$

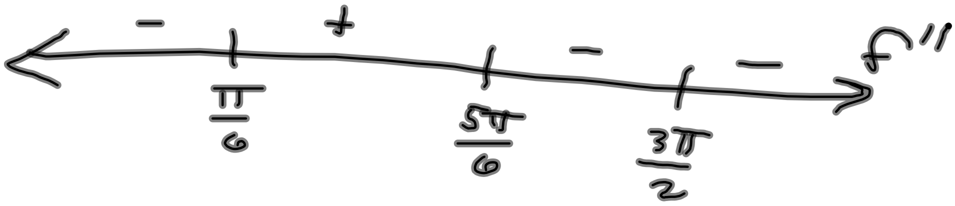


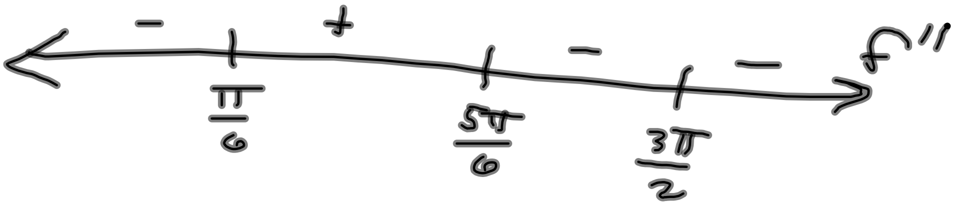
$\frac{3\pi}{2}$

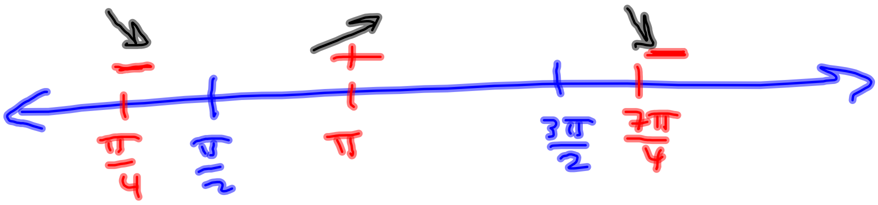


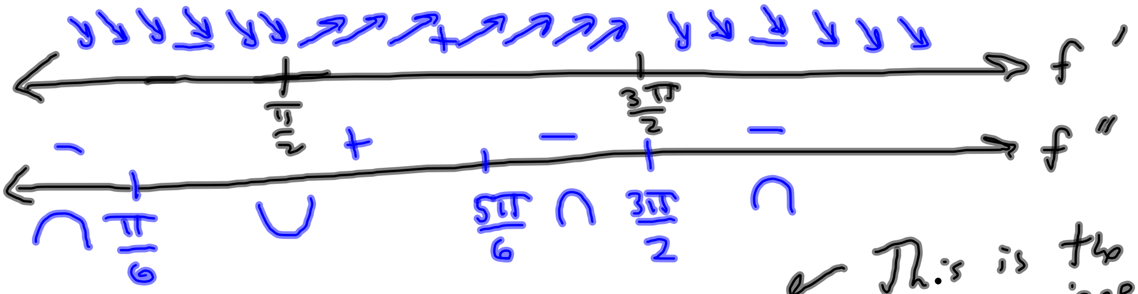
$\frac{5\pi}{6}$

Finish next time

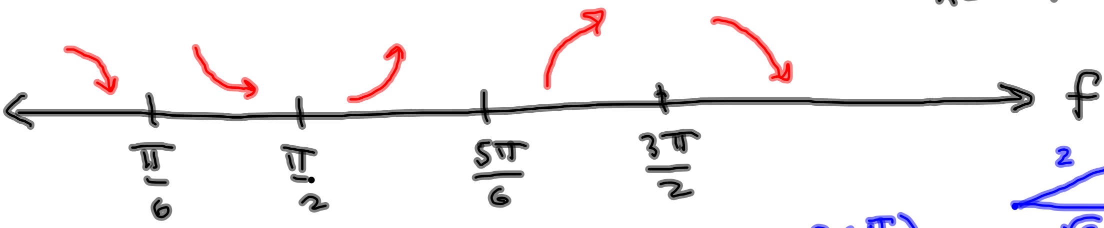


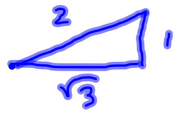






This is the new piece



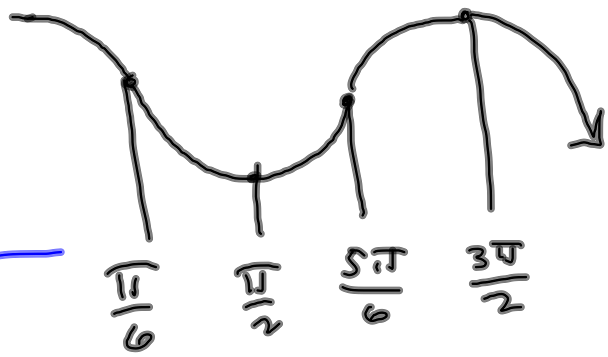


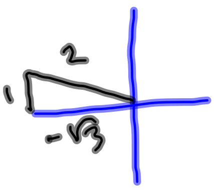
$f\left(\frac{\pi}{6}\right) =$

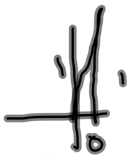
$\cos^2\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{6}\right) =$

$\frac{3}{4} - 1 = -\frac{1}{4}$

$\left(\frac{\pi}{6}, -\frac{1}{4}\right)$







$f\left(\frac{\pi}{2}\right) = 0 - 2 = -2$

$\left(\frac{\pi}{2}, -2\right)$

$f\left(\frac{5\pi}{6}\right) = \frac{3}{4} - 2\left(\frac{1}{2}\right) = -\frac{1}{4}$

$\left(\frac{5\pi}{6}, -\frac{1}{4}\right)$

$f\left(\frac{3\pi}{2}\right) = 2 \rightsquigarrow \left(\frac{3\pi}{2}, 2\right)$

Increasing: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Decreasing: $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

C-up: $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

C-down: $\left(0, \frac{\pi}{6}\right) \cup \underbrace{\left(\frac{5\pi}{6}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)}_{\left(\frac{5\pi}{6}, 2\pi\right)}$

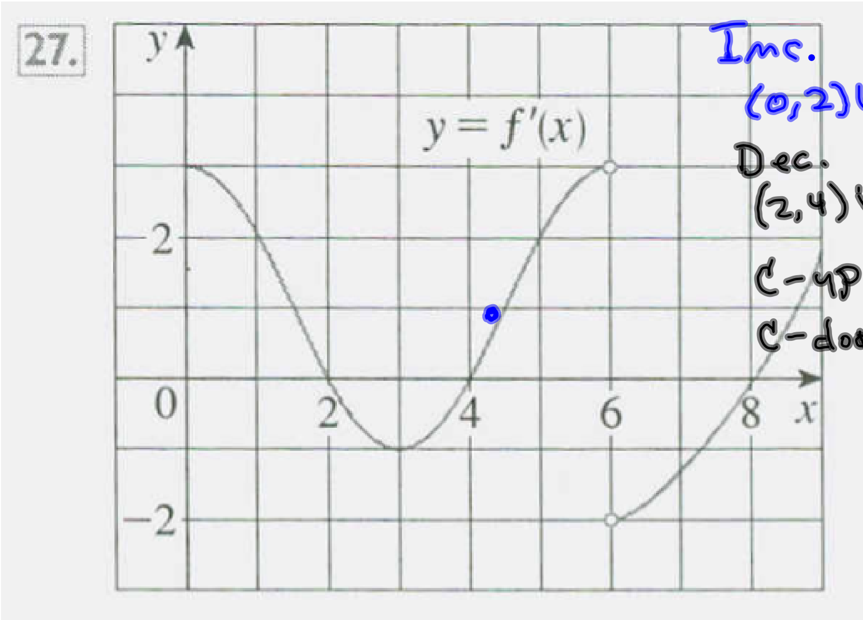
I.P.: $\left(\frac{\pi}{6}, -\frac{1}{4}\right)$, $\left(\frac{5\pi}{6}, -\frac{1}{4}\right)$

$x = \frac{3\pi}{2}$ was candidate for inflection point, but didn't pan out.

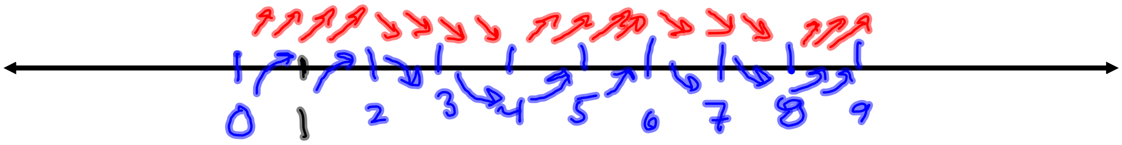
27–28 The graph of the derivative f' of a continuous function f is shown.

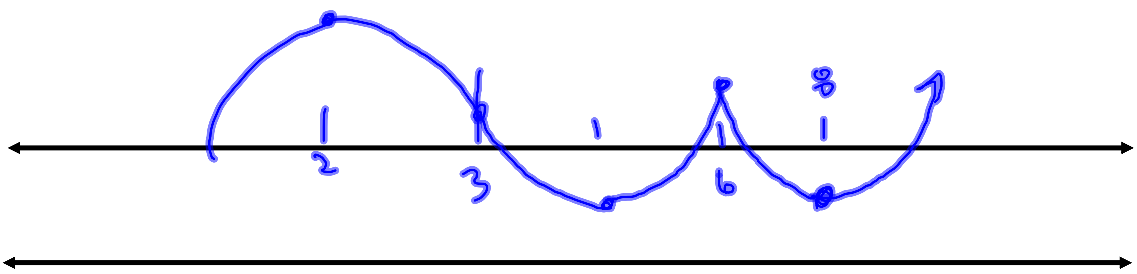
(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?
(c) On what intervals is f concave upward or downward?
(d) State the x-coordinate(s) of the point(s) of inflection.
(e) Assuming that $f(0) = 0$, sketch a graph of f.

27.



Inc.
$(0,2) \cup (4,6) \cup (8,9)$
Dec.
$(2,4) \cup (6,8)$
C-up: $(3,6) \cup (6,9)$
C-down: $(0,3)$





29–40

(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)–(c) to sketch the graph. Check your work with a graphing device if you have one.

You may want to bookmark the following link in your web browser. It can come in handy.

http://people.hofstra.edu/stefan_waner/realworld/functions/func.html

29. $f(x) = 2x^3 - 3x^2 - 12x$

56 f & g are twice-differentiable.

$f''(x) \neq 0 \ \forall x$.

(a) If f & g are concave up, show that $f+g$ is concave up.

$\hookrightarrow f''>0$ & $g''>0$

Proof

$$(f+g)'' = ((f+g)')' = (f'+g')'$$

$$= f''+g'' > 0 \text{ iff } f''>0$$

and $g''>0$, and this was given. $\blacksquare$

(b) takes more algebra.

59. Show that $\tan x > x$ for $0 < x < \pi/2$. [*Hint:* Show that $f(x) = \tan x - x$ is increasing on $(0, \pi/2)$.]