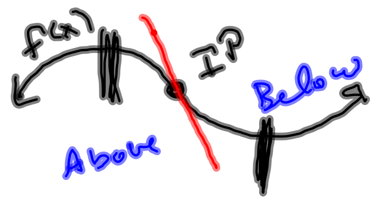
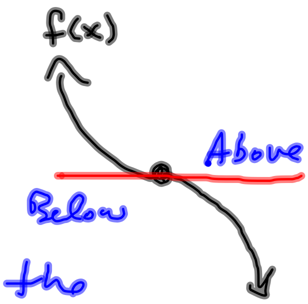
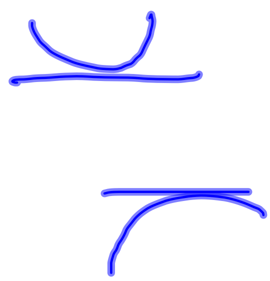
DEFINITION A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Since the second derivative is to the first derivative just as the first derivative is to the original function, what can you say about the graph of the first derivative at an inflection point?

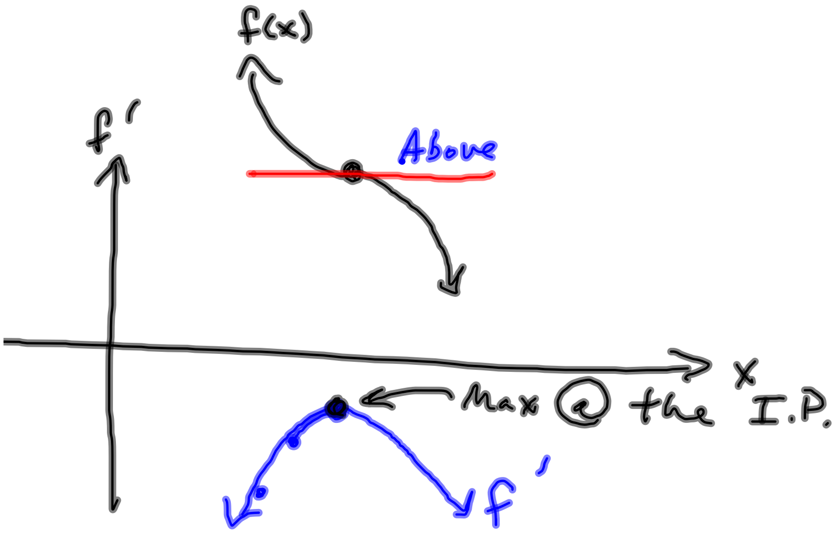






Tangent line crosses the graph at an I.P.

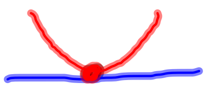
At IP, f' is max/min.

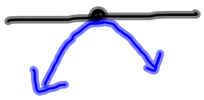


THE SECOND DERIVATIVE TEST Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

(a)  Min.

(b)  Max.

2nd Derivative Test doesn't always work. Sometimes $f''(c) = 0$, too.

#19b "Inconclusive"

2nd derivative test is cumbersome.

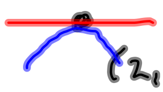
1st " " " usually preferred.

19. Suppose f'' is continuous on $(-\infty, \infty)$.

(a) If $f'(2) = 0$ and $f''(2) = -5$, what can you say about f?

(b) If $f'(6) = 0$ and $f''(6) = 0$, what can you say about f?

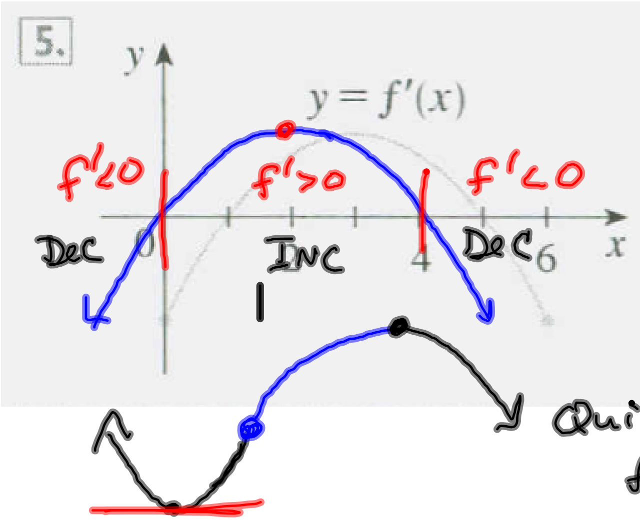
(a) Max!



$(2, f(2))$

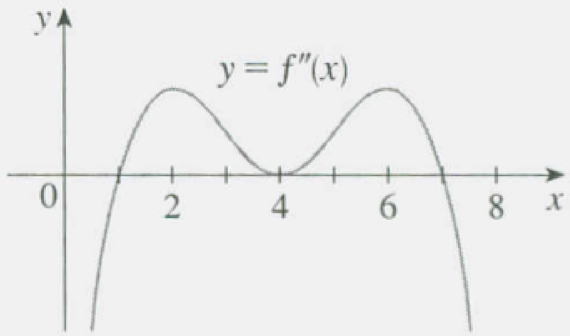
5–6 The graph of the *derivative* f' of a function f is shown.
(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?

At what values of x does f have an inflection point? $x=3$



Quick sketch of $f(x)$ from $f'(x)$.

7. The graph of the second derivative f'' of a function f is shown. State the x-coordinates of the inflection points of f. Give reasons for your answers.



Mar 4-10:24 AM

9–14

(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

We did these 3 things for #s 10 and 13. The way _I_ learned to use derivatives to graph was to analyze the sign patterns of the first and second derivatives on a number line, stack these number lines, and do a 3rd number line that incorporates the information from BOTH into the graph. Just from these analyses, I was able to "see" the graph and finish a clean version of it.

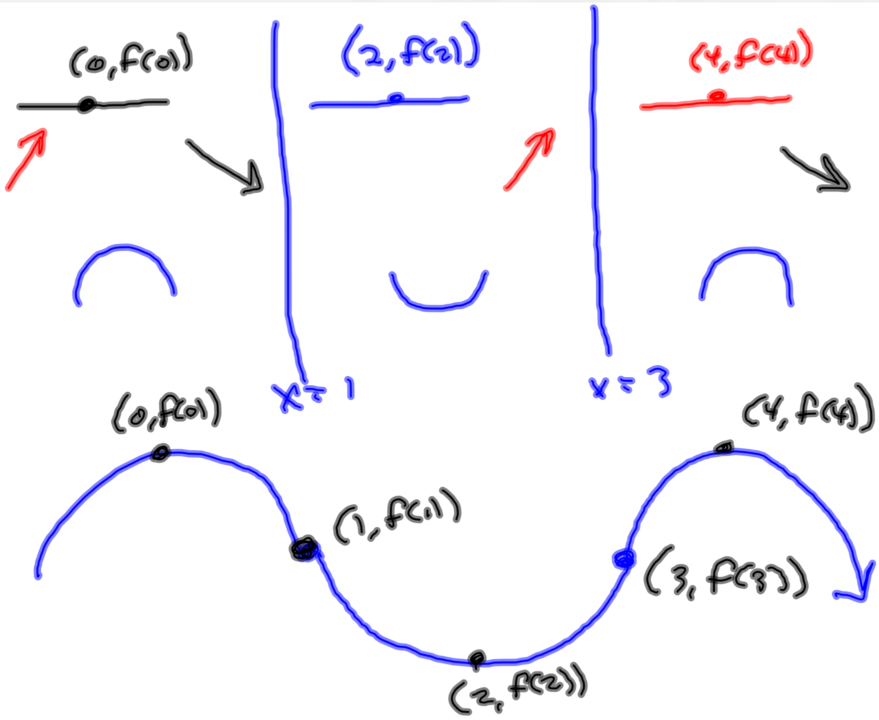
All these pieces we've been putting together are aimed at putting together graphs that show all extrema and inflection points - at really capturing the "shape" of these functions, thru calculus.

10. $f(x) = 4x^3 + 3x^2 - 6x + 1$

Mar 2-10:00 AM

20–25 Sketch the graph of a function that satisfies all of the given conditions.

21. $f'(0) = f'(2) = f'(4) = 0$,
$f'(x) > 0$ if $x < 0$ or $2 < x < 4$,
$f'(x) < 0$ if $0 < x < 2$ or $x > 4$,
$f''(x) > 0$ if $1 < x < 3$, $f''(x) < 0$ if $x < 1$ or $x > 3$



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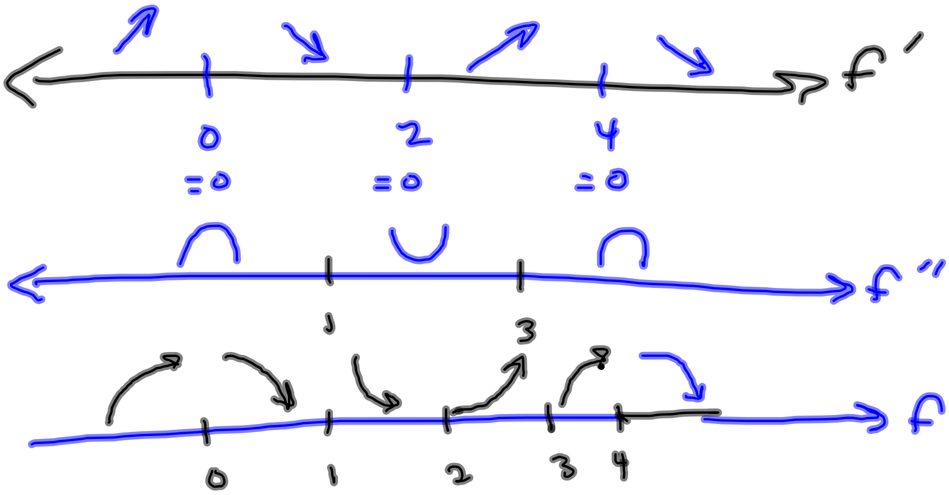


#14

$f := x \to \cos(x)^2 - 2 \cdot \sin(x)$

$$x \to \cos(x)^2 - 2\sin(x) = f(x)$$

$fp := D(f)$

$$x \to -2\cos(x)\sin(x) - 2\cos(x) = f'(x)$$

$fpp := D(fp)$

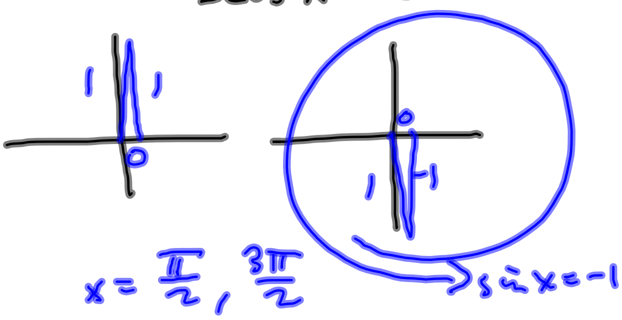
$$x \to 2\sin(x)^2 - 2\cos(x)^2 + 2\sin(x) = f''(x)$$

$f'(x) = 0 \Rightarrow \underline{-2\cos x}\,(\underline{\sin x + 1}) = 0$

$\Rightarrow -2\cos x = 0$ OR $\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

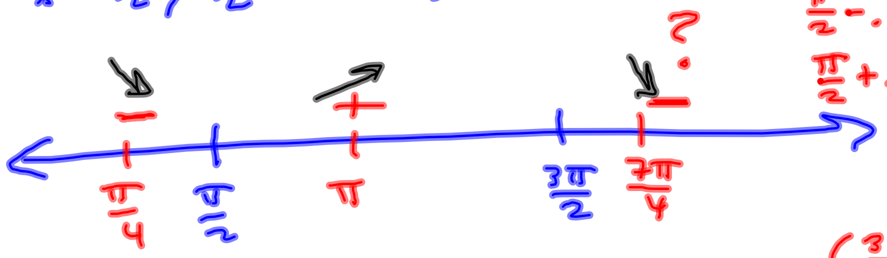


$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Test:

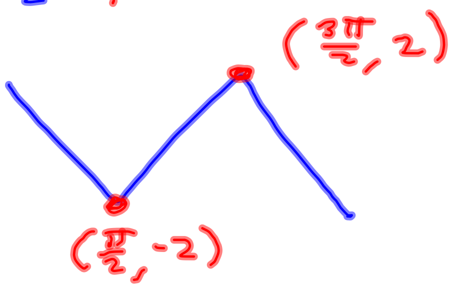
$\frac{\pi}{2} - .01$

$\frac{\pi}{2} + .01$



$f(\frac{\pi}{2}) = -2$

$f(\frac{3\pi}{2}) = 2$



$$\sin^2 x + \cos^2 x = 1$$

$$f''(x) = 2\sin^2 x - 2\cos^2 x + 2\sin x \overset{\text{SET}}{=} 0$$

$$\Rightarrow 2\sin^2 x - 2(1 - \sin^2 x) + 2\sin x$$

$$= 2\sin^2 x - 2 + 2\sin^2 x + 2\sin x$$

$$= 4\sin^2 x + 2\sin x - 2 = 0$$

$$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

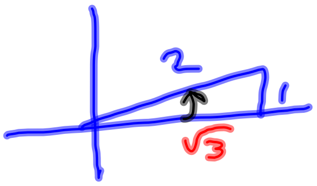
Let $u = \sin x$

$$2u^2 + u - 1 = 0$$

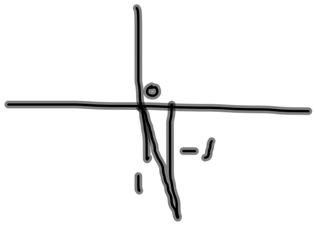
$$(2u - 1)(u + 1) = 0$$

$$u = \tfrac{1}{2} \qquad u = -1$$

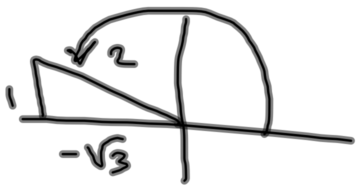
$$\sin x = \tfrac{1}{2} \quad \text{OR} \quad \sin x = -1$$



$\frac{\pi}{6}$

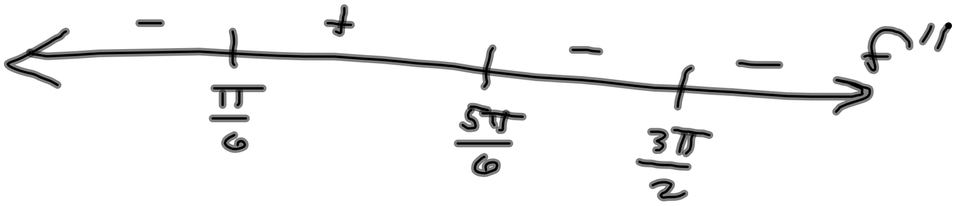


$\frac{3\pi}{2}$



$\frac{5\pi}{6}$

Finish next time



27–28 The graph of the derivative f' of a continuous function f is shown.

(a) On what intervals is f increasing or decreasing?

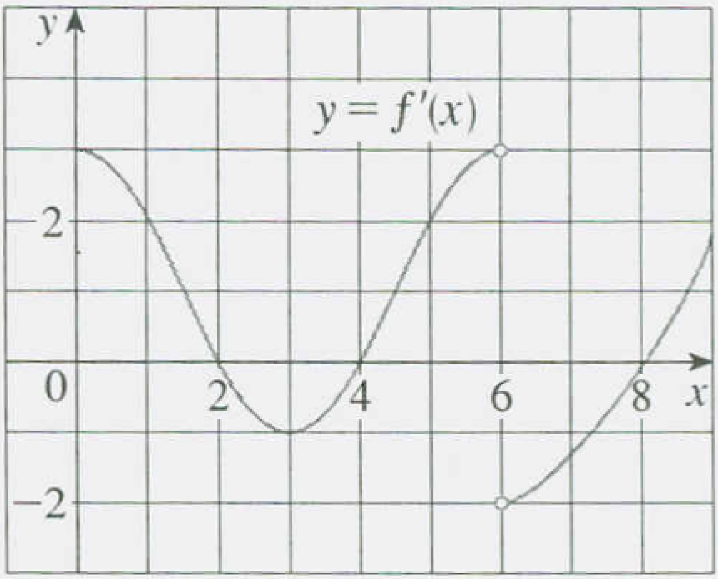
(b) At what values of x does f have a local maximum or minimum?

(c) On what intervals is f concave upward or downward?

(d) State the x-coordinate(s) of the point(s) of inflection.

(e) Assuming that $f(0) = 0$, sketch a graph of f.

27.



Mar 2-10:34 AM

29–40

(a) Find the intervals of increase or decrease.
(b) Find the local maximum and minimum values.
(c) Find the intervals of concavity and the inflection points.
(d) Use the information from parts (a)–(c) to sketch the graph. Check your work with a graphing device if you have one.

You may want to bookmark the following link in your web browser. It can come in handy.

http://people.hofstra.edu/stefan_waner/realworld/functions/func.html

29. $f(x) = 2x^3 - 3x^2 - 12x$

43–44

(a) Use a graph of f to estimate the maximum and minimum values. Then find the exact values.

(b) Estimate the value of x at which f increases most rapidly. Then find the exact value.

43. $f(x) = \dfrac{x + 1}{\sqrt{x^2 + 1}}$

59. Show that $\tan x > x$ for $0 < x < \pi/2$. [*Hint:* Show that $f(x) = \tan x - x$ is increasing on $(0, \pi/2)$.]