1–4 Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

3. $f(x) = \sqrt{x} - \frac{1}{3}x, \quad [0, 9]$

$f(0) = 0 = f(9)$ ✓

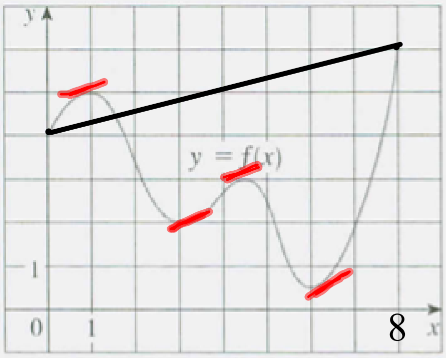
$\mathcal{D}(f) = \mathbb{R} \supset [0,9]$ ✓ f is cnt^s on $[0,9]$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

$\mathcal{D}(f') = (0,\infty) \supset (0,9)$ ✓ f is dif'b^l on $(0,9)$

5. Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

7. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval $[0, 8]$.



$x = .7, 3, 4.3, 6.1$

11–14 Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

14. $f(x) = \dfrac{x}{x+2}, \quad [1, 4]$

f is cnts on $(\mathbb{R} \setminus \{-2\}) \supseteq [1, 4]$

f is difb'l on $(\mathbb{R} \setminus \{-2\}) \supset (1, 4)$

15. Let $f(x) = (x - 3)^{-2}$. Show that there is no value of c in $(1, 4)$ such that $f(4) - f(1) = f'(c)(4 - 1)$. Why does this not contradict the Mean Value Theorem?

$$m_{avg} = \frac{f(4) - f(1)}{4 - 1} = \frac{1^{-2} - (-4)^{-2}}{3} = \frac{1 - \frac{1}{16}}{3} = \frac{\frac{15}{16}}{3} = \frac{5}{16}$$

$$f'(x) = -2(x-3)^{-3} = \frac{-2}{(x-3)^3} \stackrel{\text{SET}}{=} \frac{5}{16}$$

$$\underset{<0}{-32} = \underset{\geq 0}{5(x-3)^2}$$

Never the twain shall meet.

why no contradiction?
f ain't cnt @ $x = 3 \in [1, 4]$, so the hypotheses of the theorem do not hold.

Suppose $f(3) = 2$ and $f'(x) \leq 4$ for all $x \in [-1, 11]$. Give an upper bound for $f(6)$. (Cf #23)

Suppose $2 \le f'(x) \le 7$ for all x. Prove that $8 \le f(5) - f(1) \le 28$.
(Cf #24).

Done ✓

Intuition Guide:
I always visualize MVT in terms of slope. So, assuming the hypotheses of MVT are satisfied, I think of the conclusion this way:

$$\exists c \in (a,b) \ni f'(c) = \frac{f(b) - f(a)}{b - a}$$

But for practical purposes, and in the exercises, we often see the conclusion written this way:

$$\exists c \in (a,b) \ni f(b) - f(a) = f'(c)(b - a)$$

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26. Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$. [*Hint*: Apply the Mean Value Theorem to the function $h = f - g$.]

Homework!

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