From Syllabus:

Homework: Your final homework grade will be based on 85% of the available points (approximately 600 points available, 10 per assignment). So if you're getting 85% each assignment, on average, you will earn 100% credit for the homework segment.

Virtually every day, you will submit (well-)written homework.
Each assignment is worth 10 points. No late assignments will be accepted.

I'll accept up to end of Ch 3 late, for $\frac{5}{10}$, but that's it. I'm done.

Mar 1-12:10 PM

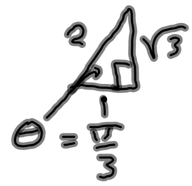
§4.1 # 42

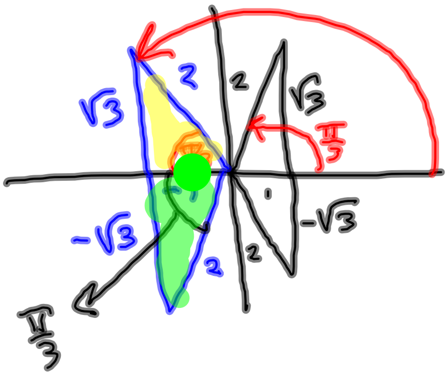
$g(\theta) = 4\theta - \tan\theta$

$g'(\theta) = 4 - \sec^2\theta = 4 - \frac{1}{\cos^2\theta}$

$g'(\theta) \overset{\text{SET}}{=} 0 \Longrightarrow \sec^2\theta = 4$

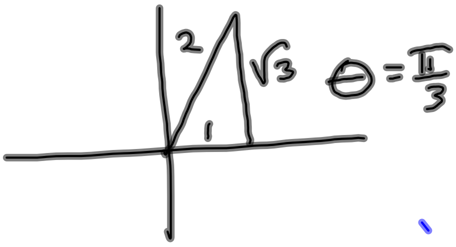
$\sec\theta = \pm 2$

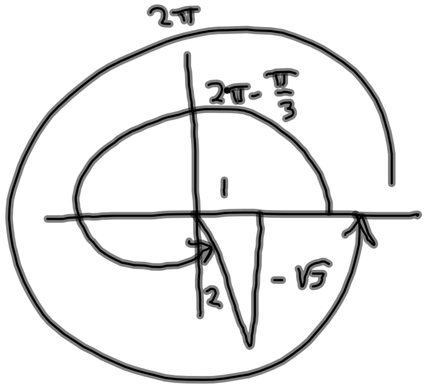




$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$





$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

On $[0, 2\pi]$, we have

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

One way: $\frac{\pi}{3} + n\pi$, $n \in \mathbb{Z}$ i.e. $(n = 0, \pm 1, \pm 2, \dots)$

$\frac{2\pi}{3} + n\pi$, $n \in \mathbb{Z}$

Easier Way:

$\frac{\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$ — All the angles that have $+\frac{\pi}{3}$ as reference angle.

$\frac{2\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$

$\frac{4\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$

$\frac{5\pi}{3} + 2n\pi$, $n \in \mathbb{Z}$

This handles $g'(\theta) = 0$

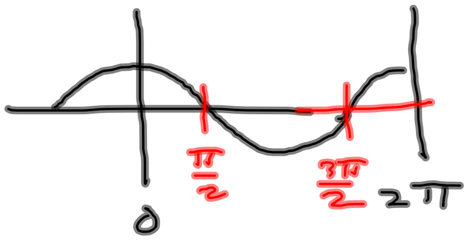
We also need to find θ where $g'(\theta) \nexists$.

When is $g'(\theta)$ undefined?

$g'(\theta) = 4 - \frac{1}{\cos^2\theta}$ is undefined when

$\cos^2\theta = 0$

$\cos\theta = 0 \Longrightarrow \theta = (2n+1)\frac{\pi}{2}, \ n \in \mathbb{Z}$



$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

But $\pm\frac{\pi}{2} + 2n\pi \notin \mathcal{D}$

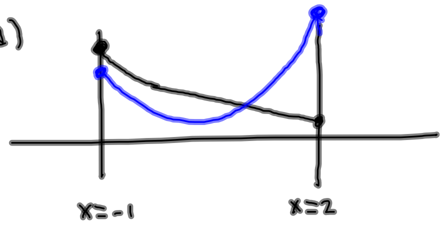
1, 3, 5, 7, 9

2n+1

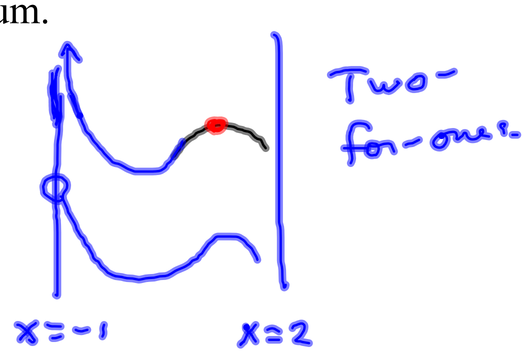
12. (a) Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no local maximum.
(b) Sketch the graph of a function on $[-1, 2]$ that has a local maximum but no absolute maximum.

Example: Sketch the graph of a continuous function on [0, 3] that has an absolute maximum, but no absolute minimum.

(a)



(b)



THE CLOSED INTERVAL METHOD To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Note that this method, given by Stewart, assumes f is continuous in the first place. By the previous example, we know/believe that critical numbers and local extrema might occur where f is not continuous.

45–56 Find the absolute maximum and absolute minimum values of f on the given interval.

48. $f(x) = x^3 - 6x^2 + 9x + 2, \quad [-1, 4]$

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 9 & 2 \\ & & -1 & 7 & -16 \\ \hline & 1 & -7 & 16 & \boxed{-14 = f(-1)} \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 9 & 2 \\ & & 4 & -8 & 4 \\ \hline & 1 & -2 & 1 & \boxed{6 = f(4)} \end{array}$$

$f'(x) = 3x^2 - 12x + 9 \overset{SET}{=} 0$

$3(x^2 - 4x + 3) = 0$

$(x-3)(x-1) = 0$

$x \in \{1, 3\}$

$f(1) = ?$

$$\boxed{\begin{array}{l} \text{Abs. Max.:} \\ \quad f(1) = f(4) = 6 \\ \text{Abs. Min:} \\ \quad f(-1) = -14 \end{array}}$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 9 & 2 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & \boxed{6 = f(1)} \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 9 & 2 \\ & & 3 & -9 & 0 \\ \hline & 1 & -3 & 0 & \boxed{2 = f(3)} \end{array}$$

Feb 26-10:46 AM

53. $f(t) = t\sqrt{4 - t^2}, \quad [-1, 2]$

$f(t) = t\,(4-t^2)^{\frac{1}{2}}$

$f(-1) = (-1)(4-(-1)^2)^{\frac{1}{2}}$

$= -1(4-1)^{\frac{1}{2}} = -\sqrt{3}$

$f(2) = 2(4-2^2)^{\frac{1}{2}} = 0$

$f'(t) = (4-t^2)^{\frac{1}{2}} + t\left(\frac{1}{2}(4-t^2)^{-\frac{1}{2}}(-2t)\right)$

$= \sqrt{4-t^2} - \dfrac{t^2}{\sqrt{4-t^2}}$

$= \dfrac{4-t^2-t^2}{\sqrt{4-t^2}} = \dfrac{4-2t^2}{\sqrt{4-t^2}} = \dfrac{2(2-t^2)}{\sqrt{4-t^2}}$

$f'(t) = 0 \Longrightarrow$

$2-t^2=0$

$\boxed{t = \pm\sqrt{2}}$

Plug in, etc.

$f'(t)$ undefined

$\sqrt{4-t^2} = 0$

$4-t^2=0$

$\boxed{t = \pm 2}$

The following problems will be presented in class by groups on Friday:

Kurt, Yi-Ling, Derek, Catherine, Johnna, ~~Josh Gagnard~~, Josh Garcia, Ken, ~~Elisha~~, Robin

S4.1

64. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \leqslant \theta \leqslant \pi/2$. Show that F is minimized when $\tan \theta = \mu$.

Silvano, Tasha, Ashley, Niloufar, Kevin, Terry, Daniel, Kelly, Andy, Beth

68. Show that 5 is a critical number of the function

$$g(x) = 2 + (x - 5)^3$$

but g does not have a local extreme value at 5.

Travis, Heather, Brigitte, David

72. A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

(a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

(b) How many local extreme values can a cubic function have?