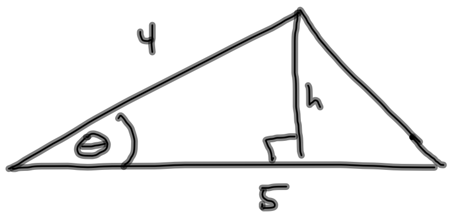


$\sin\theta = \frac{h}{4}$

$h = 4\sin\theta$

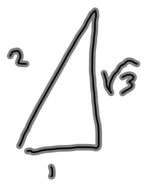
Given $\frac{d\theta}{dt} = .5 \frac{rad}{s}$

$$\begin{aligned} Area &= \tfrac{1}{2} b h \\ &= \tfrac{1}{2}(5) h \\ &= \tfrac{5}{2} h \\ &= \tfrac{5}{2}(4\sin\theta) \end{aligned}$$

5

$$\frac{d}{dt}\left[A = 10\sin\theta \right]$$

$$\frac{dA}{dt} = 10\cos\theta \frac{d\theta}{dt}$$



$$\Longrightarrow \left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}} = 10\cos\left(\frac{\pi}{3}\right)\left(.5\frac{rad}{s}\right)$$

$$= (10)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = \frac{5}{2}\ \frac{cm^2}{s}$$

③ $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

$\Rightarrow f'(1) = \frac{2}{3}$

$y = \frac{2}{3}(x-1) + 1$

④ $\sqrt[3]{(1.1)^2}$ Let $a = 1$, $f(x) = \sqrt[3]{x^2}$

$L_1(x) = \frac{2}{3}(x-1) + 1$

$= \frac{2}{3}(1.1 - 1) + 1$

$= \frac{2}{3}(.1) + 1$

$= \frac{2}{3}\left(\frac{1}{10}\right) + 1 = \frac{2}{30} + 1 = \frac{1}{15} + \frac{15}{15}$

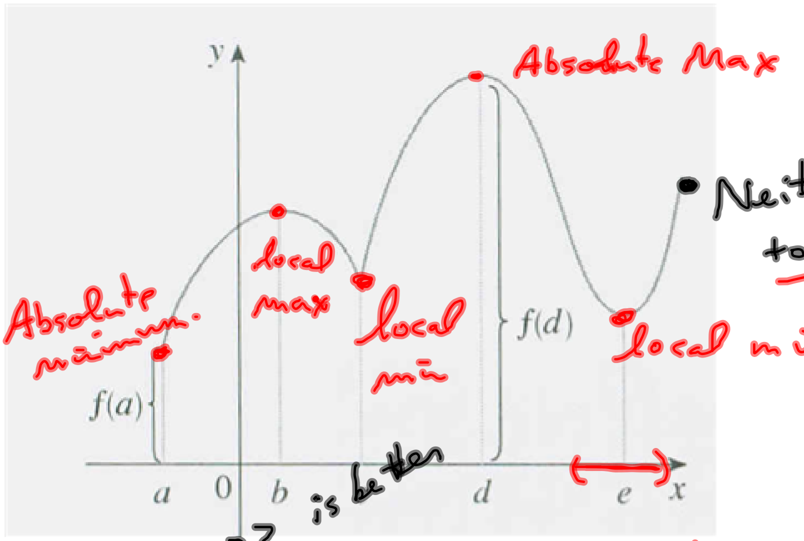
$= \frac{16}{15}$

We're scheduled for 2 days on 4.1.

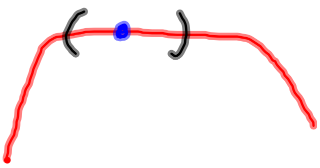
Section	Hand In	Practice Problems
4.1	#s 6, 7, 10, 13, 16, 22, 27, 33, 42, 52, 53,	Every 4th problem.

4.1 MAXIMUM AND MINIMUM VALUES

1 DEFINITION A function f has an **absolute maximum** (or **global maximum**) at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called the **maximum value** of f on D. Similarly, f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the **minimum value** of f on D. The maximum and minimum values of f are called the **extreme values** of f.



Neither. Need something to its right.



So every point on a plateau is rel. max

2 DEFINITION A function f has a **local maximum** (or **relative maximum**) at c if $f(c) \geq f(x)$ when x is near c. This means that $f(c) \geq f(x)$ for all x in some ~~open interval containing c.~~ Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ ~~when x is near c.~~

Endpoints candidates only for absolute max/min on the domain.
"Locals" need to be compared to values to their left & right.

Continuity on a **closed** interval guarantees that a function attains a global max and a global min on that interval. Extreme Value Theorem.

Things can get dicey on open intervals or if the function isn't continuous.

3 THE EXTREME VALUE THEOREM If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Nonexamples:

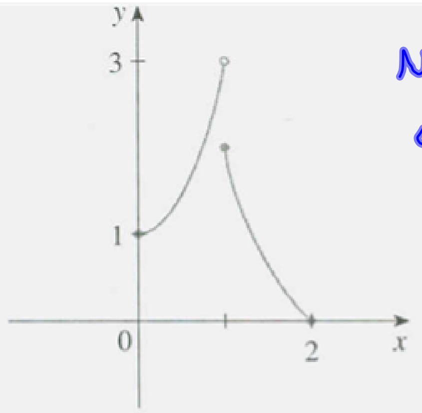


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

Not cnt's on $[0,2]$

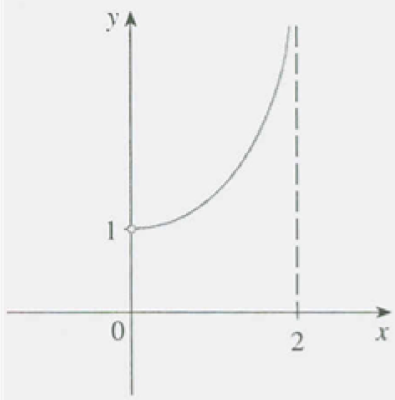


FIGURE 7
This continuous function g has no maximum or minimum.

Figure 6 DOES have a least upper bound of $y = 3$, BUT the function never quite GETS there!
Figure 6, the function is bounded above and below.

Figure 7 DOES have a greatest lower bound of $y = 1$, BUT the function never GETS there!
Figure 7, the function is bounded below, but not above.

Fermat has more than one theorem named after him. He's more "famous" in algebra. There's a Fermat's Little Theorem and Fermat's Last Theorem, to name two.

4 FERMAT'S THEOREM If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Sometimes a local extreme can occur for a continuous function where its derivative is NOT defined. Our search for extrema will include looking for places where the derivative is zero or where the derivative is undefined.

6 DEFINITION A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

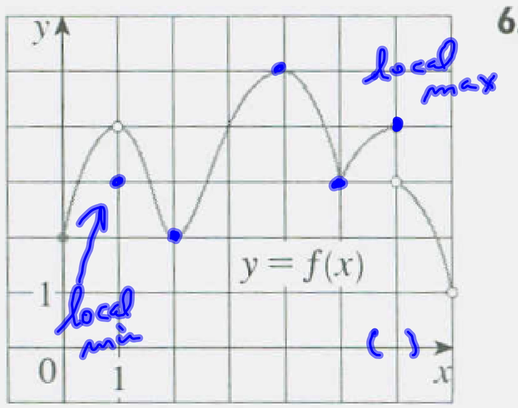
* 7 If f has a local maximum or minimum at c, then c is a critical number of f.

Note that we're not assuming that f is necessarily continuous at c, although this is usually the sorts of points we seek.

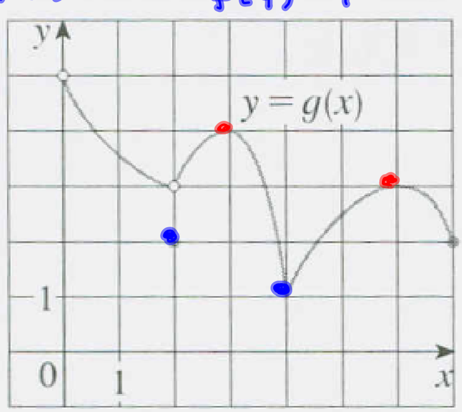
5–6 Use the graph to state the absolute and local maximum and minimum values of the function.

Abs Max: $\nexists$
Abs Min: $f(4) = 1$

5.

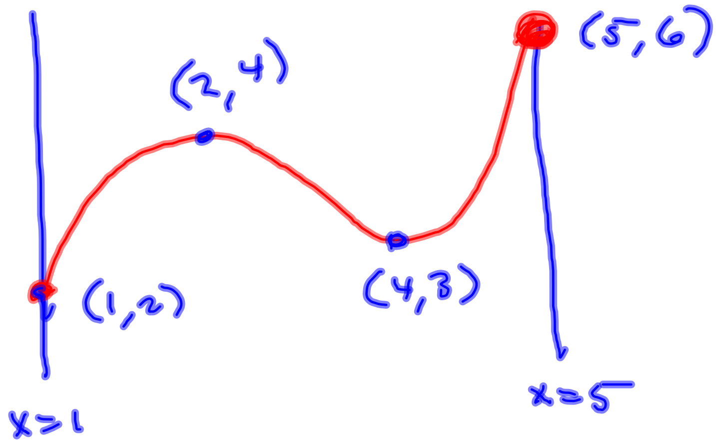


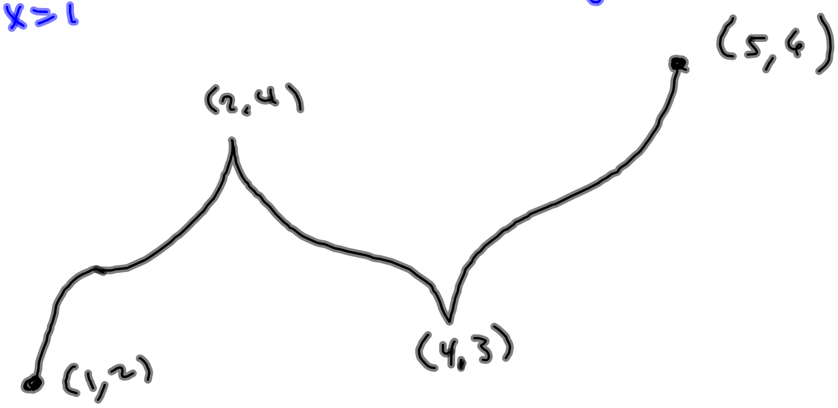
6.



7–10 Sketch the graph of a function f that is continuous on [1, 5] and has the given properties.

8. Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4





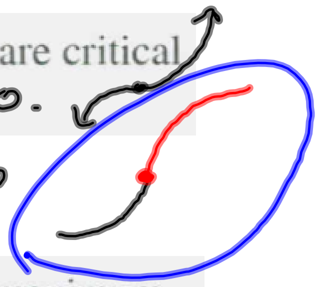
Feb 26-10:56 AM

It's conceivable that $f'(c) = 0$ or is undefined, and yet f has no max/min at $x = c$. Can you think of an example? #10 asks you to draw a picture of one. (Homework)

10. f has no local maximum or minimum, but 2 and 4 are critical numbers

Look @ x^3 when $x=0$.

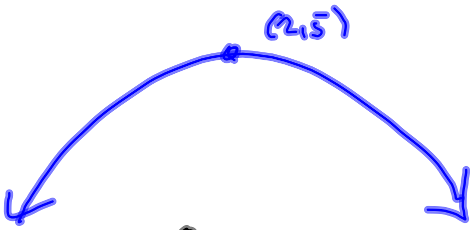
$x^{\frac{1}{3}}$ when $x=0$



#11 is kinda cute.

11. (a) Sketch the graph of a function that has a local maximum at 2 and is differentiable at 2.
(b) Sketch the graph of a function that has a local maximum at 2 and is continuous but not differentiable at 2.

(a)

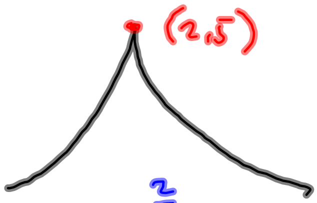


$-(x-2)^2+5$

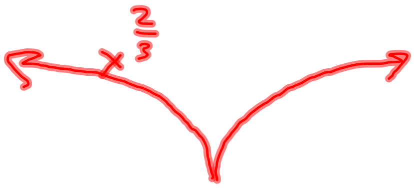
(b)

But it's not cont @ $x=2$

Try again.



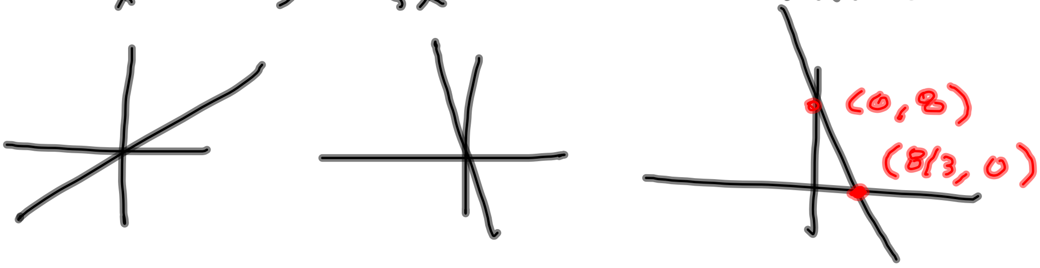
$-(x-2)^{\frac{2}{3}}+5$



15–28 Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (Use the graphs and transformations of Sections 1.2 and 1.3.)

15. $f(x) = 8 - 3x, \quad x \geqslant 1$

$x \longrightarrow -3x \longrightarrow -3x+8$



28. $f(x) = \begin{cases} 4 - x^2 & \text{if } -2 \leqslant x < 0 \\ 2x - 1 & \text{if } 0 \leqslant x \leqslant 2 \end{cases}$

29–42 Find the critical numbers of the function.

37. $h(t) = t^{3/4} - 2t^{1/4}$

$h'(t) = \frac{3}{4}t^{-\frac{1}{4}} - \frac{2}{4}t^{-\frac{3}{4}}$ $= \frac{1}{4}t^{-\frac{3}{4}}\left[3t^{\frac{1}{2}} - 2\right]$

$\frac{t^{-\frac{1}{4}}}{t^{-\frac{3}{4}}} = t^{-\frac{1}{4}-(-\frac{3}{4})}$

$= t^{+\frac{1}{2}}$

$= \frac{3}{4}\cdot\frac{1}{t^{\frac{1}{4}}} - \frac{2}{4}\cdot\frac{1}{t^{\frac{3}{4}}}$

$= \frac{3}{4}\left(\frac{1}{t^{\frac{1}{4}}}\cdot\frac{t^{\frac{2}{4}}}{t^{\frac{2}{4}}}\right) - \frac{2}{4}\left(\frac{1}{t^{\frac{3}{4}}}\right)$

$= \frac{3t^{\frac{1}{2}} - 2}{4t^{3/4}}$

$3t^{\frac{1}{2}} - 2 = 0$ or $4t^{\frac{3}{4}} = 0$

$3t^{\frac{1}{2}} = 2$ $\boxed{t = 0}$

$t^{\frac{1}{2}} = \frac{2}{3}$

$\boxed{t = \frac{4}{9}}$

Feb 26-11:06 AM

12. (a) Sketch the graph of a function on $[-1, 2]$ that has an absolute maximum but no local maximum.

(b) Sketch the graph of a function on $[-1, 2]$ that has a local maximum but no absolute maximum.

Example: Sketch the graph of a continuous function on [0, 3] that has an absolute maximum, but no absolute minimum.

THE CLOSED INTERVAL METHOD To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Note that this method, given by Stewart, assumes f is continuous in the first place. By the previous example, we know/believe that critical numbers and local extrema might occur where f is not continuous.

45–56 Find the absolute maximum and absolute minimum values of f on the given interval.

48. $f(x) = x^3 - 6x^2 + 9x + 2, \quad [-1, 4]$

53. $f(t) = t\sqrt{4 - t^2}, \quad [-1, 2]$

Feb 28-1:08 PM

The following problems will be presented in class by groups on Friday:

Kurt, Yi-Ling, Derek, Catherine, Johnna, Josh Gagnard, Josh Garcia, Ken, Elisha, Robin

64. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \leqslant \theta \leqslant \pi/2$. Show that F is minimized when $\tan \theta = \mu$.

Silvano, Tasha, Ashley, Niloufar, Kevin, Terry, Daniel, Kelly, Andy, Beth

68. Show that 5 is a critical number of the function

$$g(x) = 2 + (x - 5)^3$$

but g does not have a local extreme value at 5.

Travis, Heather, Brigitte, David

$\boxed{72.}$ A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

(a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

(b) How many local extreme values can a cubic function have?