3.9 To approximate $f(x)$ @ some x, using a "nice" x-value, $x=a$, use the tangent line!

$$y = m_{tan}(x - x_1) + y_1$$

$$y = m_{tan}(x - a) + f(a)$$

$$\boxed{f(x) \approx f'(a)(x-a) + f(a)} = L_a(x)$$

Differentials

$$dy = f'(x)\,dx$$

To approximate the change in y, Δy.

$$y = m_{tan}(x - x_1) + y_1$$

$$y = f'(a)\underbrace{(x-a)}_{\Delta x = dx} + f(a)$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\Delta y = f(x) - f(a) \approx f'(a)(x-a) = dy$$

$$\Delta y \approx f'(a)\,\Delta x$$

Will collect 3.8, 3.9 tomorrow

1. (15 pts) Find f'(x) using the definition of a derivative (the "long way") $f(x) = 3x^2 - 4x + 6$

$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-4(x+h)+6-[3x^2-4x+6]}{h}$$

$$= \frac{3(x^2+2xh+h^2)-4x-4h+6-3x^2+4x-6}{h}$$

$$= \frac{3x^2+6xh+3h^2-4x-4h+6-3x^2+4x-6}{h}$$

$$= \frac{6xh+3h^2-4h}{h} = \frac{h(6x+3h-4)}{h} = \boxed{\begin{array}{c}6x+3h-4\\ (h\neq 0)\end{array}}$$

optional step.

$$\xrightarrow{h\to 0} \boxed{6x-4}$$

Find the first derivative in problems 2 - 5. **DO NOT SIMPLIFY** your answers: (5 pts each)

2. $f(x) = 4x^5 + 3\sqrt{x} + \dfrac{3}{x^4} - 2\csc x$

$= 4x^5 + 3x^{\frac{1}{2}} + 3x^{-4} - 2\csc x$

$\Longrightarrow f'(x) = 20x^4 + \frac{3}{2}x^{-\frac{1}{2}} - 12x^{-5} + 2\csc x\cot x$

3. $f(x) = \dfrac{3x^2 - 4x}{x^5 + 5x^3 + 1}$ $\Longrightarrow$ Andy

$$f'(x) = \frac{(6x-4)(x^5+5x^3+1) - (3x^2-4x)(5x^4+15x^2)}{(x^5+5x^3+1)^2}$$

$f(x) = (3x^2-4x)(x^5+5x^3+1)^{-1}$ Johna's way

$f'(x) = (6x-4)(x^5+5x^3+1)^{-1}$

$+ (3x^2-4x)(-1(x^5+5x^3+1)^{-2}(5x^4+15x^2))$

l

4. $f(x) = \tan(\sin x + \sec x)$

$$\Longrightarrow f'(x) = \left(\sec^2(\sin x + \sec x)\right)(\cos x + \sec x \tan x)$$

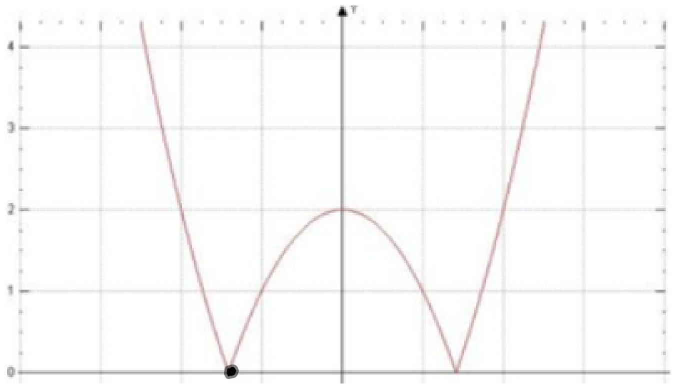
5. $f(x) = 5x^3 \sqrt{x^4 - 2x + 7}$

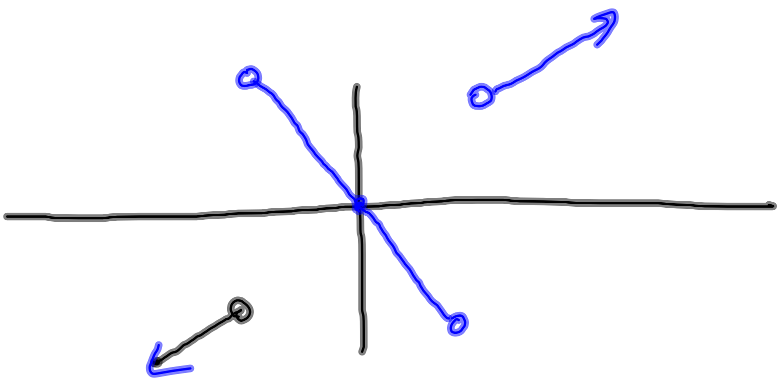
$$= 5x^3 (x^4 - 2x + 7)^{\frac{1}{2}}$$

$$\Longrightarrow f'(x) = 15x^2 (x^4 - 2x + 7)^{\frac{1}{2}} + 5x^3 \left(\tfrac{1}{2} (x^4 - 2x + 7)^{-\frac{1}{2}} (4x^3 - 2)\right)$$

6. (10 pts) The graph of function f is given here.

Use slopes of tangents to sketch the graph of f'.



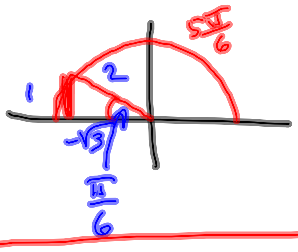


7. (15 pts) Find the equation of the line tangent to the curve $y = \sin^2 x$ at the point $(5\pi/6,\ 1/4)$

$f(x) = \sin^2(x)$ $\quad f'(x) = 2\sin(x)\cos x$

$f'\left(\frac{5\pi}{6}\right) = 2\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{5\pi}{6}\right)$

$= 2\left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$



$y = m_{tan}(x - x_1) + y_1$

$y = -\frac{\sqrt{3}}{2}\left(x - \frac{5\pi}{6}\right) + \frac{1}{4}$

$\frac{8\pi}{9}$

$\frac{5\pi}{6} + \frac{\pi}{18} = \frac{15\pi + \pi}{18} = \frac{16\pi}{18}$

$= \frac{8\pi}{9}$.

Approximate $f\left(\frac{8\pi}{9}\right)$ Adding a question

$a = \frac{5\pi}{6}$, $x = \frac{8\pi}{9}$

Plug $x = \frac{8\pi}{9}$ into $L(x)$!

$f\left(\frac{8\pi}{9}\right) \approx$

$f\left(\frac{5\pi}{6}\right) + f'\left(\frac{5\pi}{6}\right)\left(x - \frac{5\pi}{6}\right)$

$= \frac{1}{4} - \frac{\sqrt{3}}{2}\left(\frac{8\pi}{9} - \frac{5\pi}{6}\right)$

$= \frac{1}{4} - \frac{\sqrt{3}}{2}\left(\frac{\pi}{18}\right)$

8. (15 pts) Find dy/dx: $2xy^3 + 5y^2 = 3y - x^3$

$2y^3 + 2x(3y^2y') + 10yy' = 3y' - 3x^2$

$6xy^2y' + 10yy' - 3y' = -3x^2 - 2y^3$

$y'(6xy^2 + 10y - 3) = -3x^2 - 2y^3$

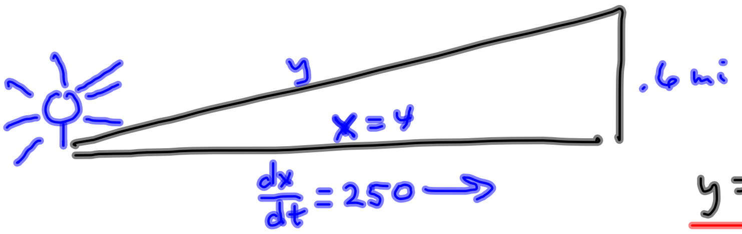
$y' = \frac{-3x^2 - 2y^3}{6xy^2 + 10y - 3}$

$$\frac{d}{dx}\left[(f(x))^2\right] = 2f(x)\cdot f'(x)$$

$$\frac{d}{dx}\left[y^2\right] = 2yy'$$

9. (15 pts) A plane flying horizontally at an altitude of 0.6 miles and a speed of 250 miles/hour passes directly over a radar station. Find the rate at which the distance from the plane to the station (the diagonal distance) is increasing when the plane is directly over a spot on the ground that is 4 miles away (horizontally) from the station.

Want $\left.\frac{dy}{dt}\right|_{x=4}$



$$y = L_a(x) = f'(a)(x-a) + f(a)$$

$$y^2 = x^2 + (.6)^2$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt} \quad \text{Let } x=4 \Longrightarrow$$

$$2y\frac{dy}{dt} = 2(4)\cdot(250)$$

Need y:

$$y^2 = 4^2 + .36 = 16.36$$

$$y = \pm\sqrt{16.36}$$

Take $y > 0$:

$$y = \sqrt{16.36}$$

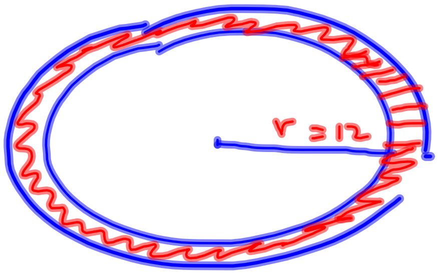
$$2\sqrt{16.36}\,\frac{dy}{dt} = 8(250) = 2000$$

$$\Longrightarrow \frac{dy}{dt} = \frac{2000}{2\sqrt{16.36}}$$

$$= \frac{1000}{\sqrt{16.36}} \approx 247.2340882 \approx 247.2\,\frac{mi}{hr}$$

Feb 24-8:08 AM

10. (10 pts) The radius of a circle is measured as 12 cm with a possible error in measurement of 0.2 cm. Use differentials to estimate the maximum possible error in computing the area of the circle.



err

$A = \pi r^2$

$dA = 2\pi r \, dr \approx \Delta A$

let $r = 12$, $dr = \Delta r = .2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r \, dr$

error

$\Delta A \approx dA = 2\pi(12)(.2)$

$= 4.8\pi \text{ cm}^2 \approx 15.0796$

$A(12.1) - A(12) = \Delta A$ actual $= \pi(12.2)^2 - \pi(12)^2 \approx 15.2053$