

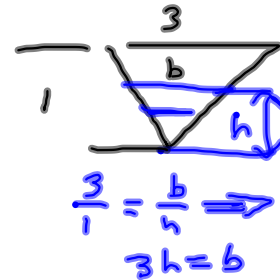
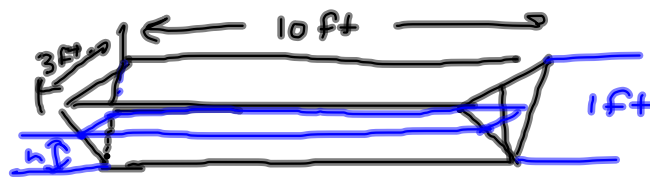
$$\sqrt{(\cos x - 5)^2} \sin x$$

$$= |\cos x - 5| \sin x$$

$$= (5 - \cos x) \sin x$$

$$\sqrt{x^2} = |x|$$

3.8 #24



$$\frac{dV}{dt} = 12 \frac{\text{ft}^3}{\text{min}}$$

$$\text{Find } \frac{dh}{dt} \Big|_{h=6''=.5\text{ft}}$$

$$\begin{aligned} V &= \frac{1}{2}bh \cdot 10 \\ &= \frac{1}{2} \cdot 3h^2 \cdot 10 \\ &= 15h^2 \end{aligned}$$

$$V = 15h^2$$

$$\frac{dh}{dt} \Big|_{h=.5} = \frac{4}{5} \text{ ft/min.}$$

$$12 = \frac{dV}{dt} \Big|_{h=\frac{1}{2}} = 30h \frac{dh}{dt} \Big|_{h=\frac{1}{2}} = 30\left(\frac{1}{2}\right) \frac{dh}{dt}$$

$$\begin{aligned} 2z z' &= 2x x' + 2y y' \\ &= 2(5)(2) + 2(12)(3) \end{aligned}$$

Verify that $L_0(x) = -8x + 1$ for

$$f(x) = (2x+1)^{-4}$$

$$f'(x) = -4(2x+1)^{-5}(2) = -\frac{8}{(2x+1)^5} \Rightarrow$$

$$f'(0) = -8 = m_{tan} \Rightarrow$$

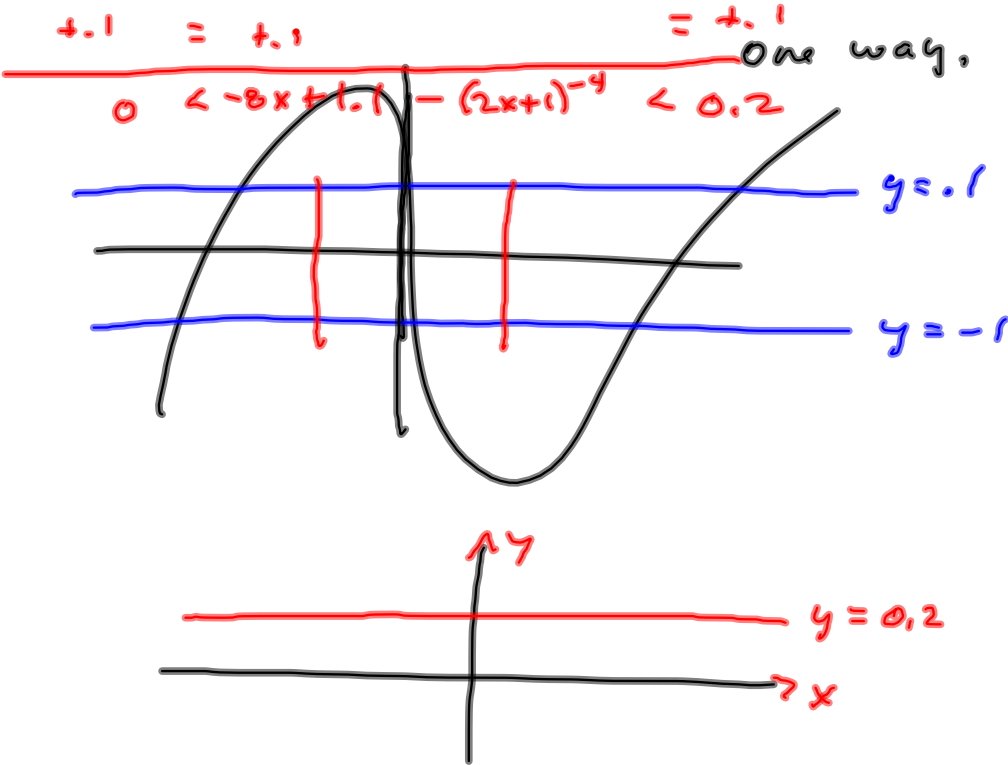
$$y = L_0(x) = -8(x-0) + f(0) \\ = -8x + 1 \quad \checkmark$$

What range of values will keep $L_0(x)$ within 0.1 units of $f(x)$ itself?

WANT

$$|-8x+1 - (2x+1)^{-4}| < .1$$

$$-.1 < \underbrace{-8x+1 - (2x+1)^{-4}}_{G(x)} < .1$$



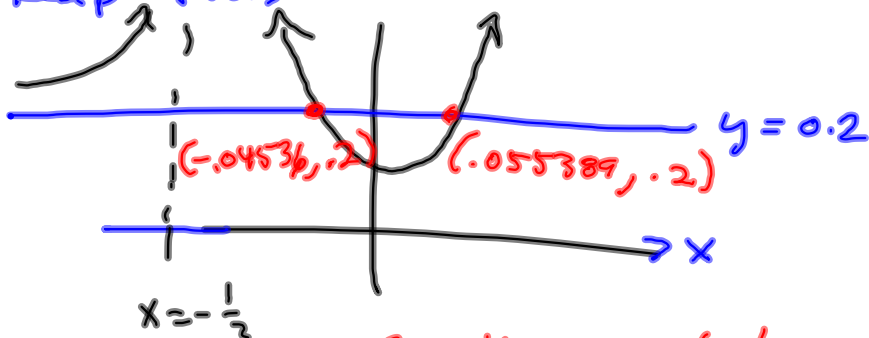
Here's how I did it.

$$\left| \frac{1}{(1+2x)^4} - (1-8x) \right| < .1$$

$$-.1 < \frac{1}{(1+2x)^4} - 1 + 8x < .1$$

$$0 < \frac{1}{(1+2x)^4} - .9 + 8x < .2$$

Keep this $\uparrow$ between 0 & .2.



So keep x between
 $-.045$ & $+.055$ & you'll
 be OK.

$$(5 - \cos x)^2 = (\cos x - 5)^2$$

$$(1 + 2x)^3 = (2x + 1)^3$$

$$* \quad \frac{L_2(x) = f(a) + f'(a)(x-a) \approx f(x)}{\sqrt{98}} \quad \downarrow \text{to approximate } f(x)$$

$$\Delta y = f(x) - f(a)$$

$$* \quad \frac{dy = f'(a)(x-a) \approx \Delta y}{\downarrow \text{To approximate } \Delta y}$$

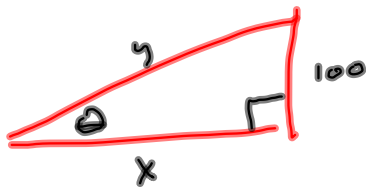
#6 on 3.9 sucked, in hindsight. $\sqrt[3]{1+x}$
 To approximate $\sqrt[3]{.95}$, isn't it
 easier to use $g(x) = \sqrt[3]{x}$ & $a = 1$

Version of #6

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \Rightarrow f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$$

$$a = 1, \text{ then } f(a) = 1, f'(a) = \frac{1}{3}(1)^{-\frac{2}{3}} = \frac{1}{3}$$

$$\begin{aligned} f(x) &\approx f(a) + f'(a)(x-a) \\ &= 1 + \left(\frac{1}{3}\right)(.95 - 1) \\ &= 1 + \left(\frac{1}{3}\right)(-.05) \\ &= 1 - \left(\frac{1}{3}\right)\left(\frac{5}{100}\right) = 1 - \left(\frac{1}{3}\right)\left(\frac{1}{20}\right) \\ &= \frac{60-1}{60} = \frac{59}{60} \approx \sqrt[3]{.95} \end{aligned}$$



$$\tan \theta = \frac{100}{x}$$

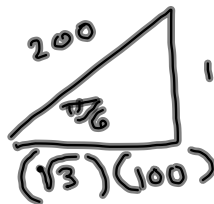
$$\sec^2 \theta \frac{d\theta}{dt} = -100x^{-2} \frac{dx}{dt}$$

~~$$\left(\frac{\sqrt{3}}{2}\right)^2 \frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2} \cdot 8$$~~

~~$$\frac{3}{4} \frac{d\theta}{dt} = -\frac{1}{300} \cdot 8$$~~

~~$$\frac{d\theta}{dt} = -\frac{8}{300} \cdot \frac{4}{3}$$~~

~~$$= -\frac{32}{900}$$~~



~~$$= -\frac{16}{450} = -\frac{8}{225} =$$~~

$$\frac{dx}{dt} = 8 \frac{ft}{s}$$

$$\text{want } \frac{d\theta}{dt} \Big|_{y=200}$$

Scratch

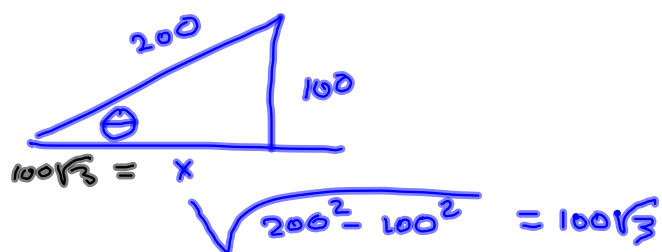
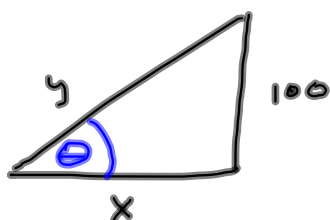
$$y = 200 \Rightarrow$$

$$x = \sqrt{200^2 - 100^2}$$

$$= \sqrt{(200-100)(200+100)}$$

$$= \sqrt{100(300)}$$

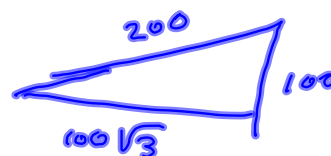
$$= \sqrt{30000} = 100\sqrt{3}$$



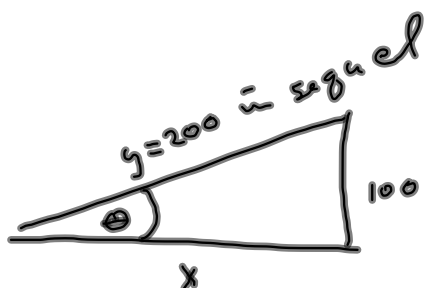
$$\cot \theta = \frac{x}{100}$$

$$-\csc^2 \theta \frac{d\theta}{dt} = \frac{1}{100} \frac{dx}{dt}$$

$$-4 \frac{d\theta}{dt} = \frac{1}{100} \frac{dx}{dt}$$



$$\frac{d\theta}{dt} = -\frac{1}{400} \cdot 8 = -\frac{2}{100} = -\frac{1}{50}$$



$$x = 100\sqrt{3}, \text{ when } y=200$$

$$\tan \theta = \frac{100}{x}$$

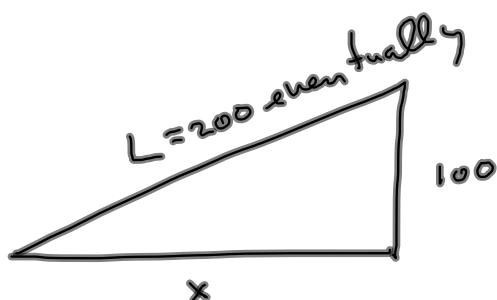
$$\sec^2 \theta \frac{d\theta}{dt} = -\frac{100}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{x}{y} = \frac{100\sqrt{3}}{200} = \frac{\sqrt{3}}{2}$$

$$\left(\frac{2}{\sqrt{3}}\right)^2 \frac{d\theta}{dt} = -\frac{100}{(100\sqrt{3})^2} \cdot 8$$

$$\frac{4}{3} \cdot \frac{d\theta}{dt} = -\frac{1}{100 \cdot 3} \cdot 8$$

$$\frac{d\theta}{dt} = -\frac{\overset{2}{\cancel{8}}}{\underset{100}{\cancel{300}}} \cdot \frac{\overset{1}{\cancel{3}}}{\cancel{4}} = -\frac{2}{100} = -\frac{1}{50}$$



$$\frac{d}{dt} \left[\frac{x}{L} \right] = \frac{x' L - x L'}{L^2}$$

$$\frac{100\sqrt{3}}{200} = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{100}{x}$$

$$\cot \theta = \frac{x}{100}$$

$$\sec^2 \theta \frac{d\theta}{dt} = -\frac{100}{x^2} \cdot \frac{dx}{dt}$$

$$-\csc^2 \theta = \frac{1}{100} \frac{dx}{dt}$$

200