3.7 #26 ?

(a) (ii) $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \frac{1}{2L}\frac{\sqrt{T}}{\sqrt{\rho}} = \frac{1}{2L\sqrt{\rho}}T^{\frac{1}{2}}$

$$\boxed{\frac{df}{dT} = \frac{1}{2L\sqrt{\rho}}\cdot\frac{1}{2}T^{-\frac{1}{2}}} = \frac{1}{4L\sqrt{\rho}}\cdot\frac{1}{\sqrt{T}} = \frac{1}{4L\sqrt{\rho T}}\cdot\frac{\sqrt{\rho T}}{\sqrt{\rho T}}$$

$$= \frac{\sqrt{\rho T}}{4L\rho T}$$

① $\frac{d}{dx}[\sin x] =$

② $\frac{d}{dx}[\cos x] =$

③ $\frac{d}{dx}[\tan x] =$

④ $\frac{d}{dx}[\sec x] =$

⑤ $\frac{d}{dx}[\csc x] =$

⑥ $\frac{d}{dx}[\cot x] =$

Separate Sheet

① $\cos x$

② $-\sin x$

③ $\sec^2 x$

④ $\sec x \tan x$

⑤ $-\csc x \cot x$

⑥ $-\csc^2 x$

⑦ $(fg)' = f'g + fg'$

⑧ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y = m(x - x_1) + y_1$$

$$= \underbrace{y_1}_{\text{ORIGINAL HEIGHT}} + \underbrace{m}_{\text{Rate of climb}}\underbrace{(x - x_1)}_{\text{Change in } x}$$

Tangent line version

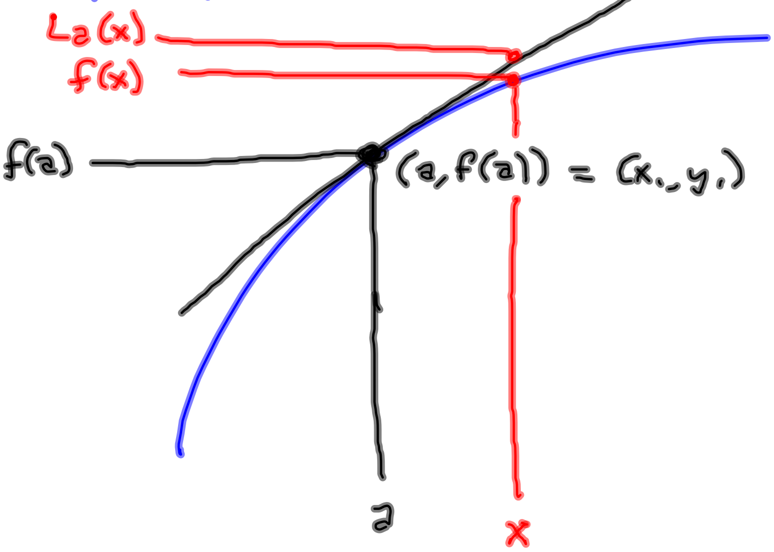
$$y = f(x_1) + f'(x_1)(x - x_1)$$

Book says Let $L_a(x)$ be the tangent line of f @ $x = a$, i.e.

$$L_a(x) = f(a) + f'(a)(x - a)$$

Also called the Linearization of f @ a.

We use it to approximate $f(x)$ close to $x = a$



Handy for figuring $\sqrt{95}$, using $\sqrt{100} = 10$ & some calculus.

$L_a(x) \approx f(x)$ close to $x = a$.

Approximate $\sqrt{95}$ with this:

$f(x) = \sqrt{x}$, $a = 100$. We want

$$\sqrt{95} \approx \sqrt{100} + \frac{d}{dx}\left[\sqrt{x}\right]\Big|_{x=100}(95-100)$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \Longrightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f(a) = f(100) = \sqrt{100} = 10 = f(a)$$

$$f'(a) = f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2 \cdot 10} = \frac{1}{20} = f'(a)$$

$$\sqrt{95} = f(95) \approx f(100) + f'(100)(95-100)$$

$$= \sqrt{100} + \frac{1}{20}(-5)$$

$$= 10 - \frac{5}{20} = 10 - \frac{1}{4} = \frac{39}{4} = 9.75$$

$$\sqrt{95} \approx 9.746794345$$

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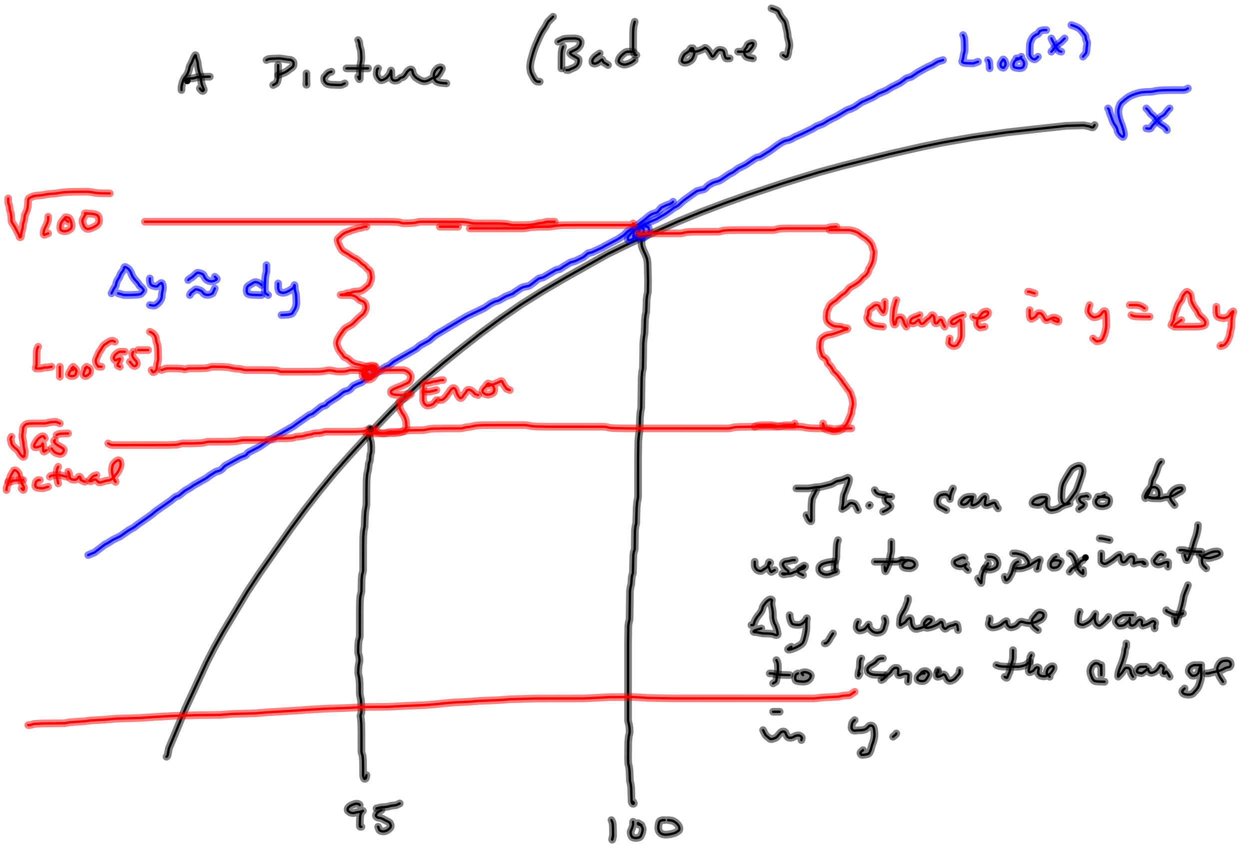
A Picture (Bad one)



This can also be used to approximate Δy, when we want to know the change in y.

This can also be used to approximate Δy, when we want to know the change

$$y = y_1 + m(x - x_1)$$

$$y = y_1 + f'(x_1)(x - x_1) \Longrightarrow$$

$$\underbrace{y - y_1}_{\Delta y \text{ on the tangent line}} = f'(x_1)(x - x_1)$$

$$\Delta x = x - x_1$$

$$\Delta y = f'(x_1)\Delta x$$

$$\frac{\Delta y}{\Delta x} = f'(x_1)$$

differentials :

$$\frac{dy}{dx} = f'(x)$$

$$dy = f'(x)\,dx$$

Differential of y.

Pretend $\frac{dy}{dx}$ is a fraction

$$dy \approx \Delta y$$

$$\underbrace{f'(x_0)\,dx}_{\text{Change in the tangent line.}} \approx \underbrace{f(x) - f(x_0)}_{\text{Actual change in the function}}$$

We did something that hinted at what we're doing NOW, when we looked at 3.7 #16 in class the other day.

Now we're ready to do things like:

"Approximate the change in volume of the sphere, when its radius changes from 5 micrometers to 5.1 micrometers."

This is very much like finding the surface area and then just multiplying by the thickness of the layer (0.1 micrometers) that we added to it. This might be a good visual for you.

$$V(r) = \frac{4}{3}\pi r^3 = V$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\Delta V \approx dV$$

$$\Delta r = dr = .1$$

$$dV = 4\pi r^2\, dr$$

$$\Delta V \approx (4\pi(5)^2)(.1) \approx 10\pi \approx 31.4159$$

Actual

$$\Delta V = \frac{4}{3}\pi(5.1)^3 - \frac{4}{3}\pi(5)^3$$

$$= \frac{4}{3}\pi\left[5.1^3 - 5^3\right] \approx 32.0484$$

Fairly close.

$V = \frac{4}{3}\pi r^3$ w.r.t. r

$\frac{dV}{dr} = 4\pi r^2$ w.r.t. t.

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ Chain Rule on the r^2

$r = r(t)$ implicitly

So $r^2 = (r(t))^2$

$f(r(t))$, when $f(*) = *^2$

$(f(r(t)))' = f'(r(t)) \cdot r'(t)$

$f(x) = x^2$ $f'(x) = 2x$

$f(r(t)) = (r(t))^2$ $\underbrace{f'(r(t)) = 2r(t)}_{\frac{df}{dr}}$.

$$\frac{d}{dx}\left[\left(\sin(x^2)\right)^{13}\right]$$

$$= 13\left(\sin(x^2)\right)^{12}\left(\cos(x^2)\right)(2x)($$

$$\frac{d}{dx}[x] = \frac{dx}{dx} = 1$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$\frac{d}{dt}[R_1] = .3 \frac{\Omega}{s}$

Implies $R_1 = R_1(t)$ is to be treated as a function of t for the purpose of differentiation.

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