3.6 #33

Wants y''

$$9x^2+y^2=9$$

$$18x+2yy'=0$$

$$y'=-\frac{9x}{y}$$

$$y''=\frac{-9y-(-9x)(y')}{y^2}$$

$$=\frac{-9y+9x\left(-\frac{9x}{y}\right)}{y^2}$$

$$=\frac{\frac{-9y^2-81x^2}{y}}{y^2} =$$

3.7 #20

$$18+2y'y'+2yy''=0$$

$$18+2(y')^2+2y\,y''=0$$

$$y''=\frac{-18-2(y')^2}{2y}$$

$$=\frac{-18-2\left(\frac{81x^2}{y^2}\right)}{2y}$$

$$=\frac{-9-\frac{81x^2}{y^2}}{y}$$

$$=\frac{-9y^2-81x^2}{y^3}$$

3.7 #20

Treat G, m, M as constants.

#32 – Bonus 5 pts

"Stable population"

means $\frac{dP}{dt} = 0$ according to the book.

3.7 #27 Marginal Cost $= C'(x) \approx C(x+1) - C(x)$

Cost of 201^{st} yard of fabric

$= C(201) - C(200) \approx C'(200)$

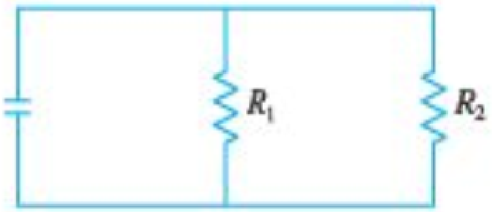
§3.8 Work thru the examples,

33. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Assume R, R_1, R_2 are functions of t.

If R_1 and R_2 are increasing at rates of 0.3 Ω/s and 0.2 Ω/s, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



We want

$$\left.\frac{dR}{dt}\right|_{\substack{R_1 = 80\,\Omega \\ R_2 = 100\,\Omega \\ R_1' = .3\,\Omega/s \\ R_2' = .2\,\Omega/s}}$$

$$R^{-1} = R_1^{-1} + R_2^{-1} \implies$$

$$-R^{-2}R' = -R_1^{-2}\cdot R_1' - R_2^{-2}\cdot R_2' \quad \text{by Chain Rule}$$

$$\left.-R^{-2}R'\right|_{\substack{R_1 = 80\,\Omega \\ R_2 = 100\,\Omega \\ R_1' = .3\,\Omega/s \\ R_2' = .2\,\Omega/s}} = -(80)^{-2}(.3) - (100)^{-2}(.2)$$

Want R' Need R.

Scratch:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

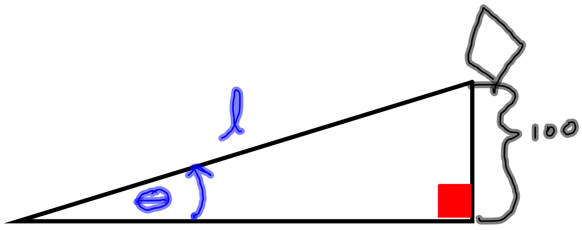
$$\frac{1}{R} = \frac{1}{80}\cdot\frac{5}{5} + \frac{1}{100}\cdot\frac{4}{4} = \frac{9}{400}$$

$$\implies R = \frac{400}{9}$$

$$\implies R' = \frac{-(80)^{-2}(.3) - (100)^{-2}(.2)}{-\left(\frac{400}{9}\right)^{-2}}$$

$\boxed{\text{etc.}}$

28. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?



$\frac{dx}{dt} = 8 \frac{ft}{s}$

want $\left.\frac{d\theta}{dt}\right|_{\ell=200}$

Want to relate θ to x, since $\frac{dx}{dt}$ is given.

$\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt}$

$\cos\theta = \frac{x}{\ell} = \frac{x}{\sqrt{100-x^2}}$

$\tan\theta = \frac{100}{x}$

$\cot\theta = \frac{x}{100}$

These will not have $\frac{d\theta}{dt}$ determined. But that's the point.

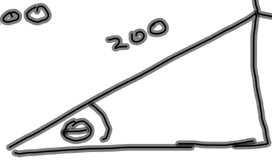
Differentiate w.r.t. 't'.

$-\csc^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{100} \cdot \frac{dx}{dt}$

By Chain Rule

Test 2 chapter 3 Friday

want $\left.\frac{d\theta}{dt}\right|_{\ell=200}$



My Goal.

This gives

$-\csc^2\left(\frac{\pi}{6}\right) \cdot \frac{d\theta}{dt} = \frac{8}{100}$

$\theta = \operatorname{Arcsin}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$-(\sqrt{3})^2 \cdot \frac{d\theta}{dt} = \frac{2}{25}$

$\frac{d\theta}{dt} = -\frac{2}{25} \cdot \frac{1}{3} = -\frac{2}{75}$

2, 1, $\sqrt{3}$

Radians per second.

3.8 Relate the variable you want to the one(s) whose rate is given.

Analytic Geometry & Trig in this section.

3.4 #18

Typo by teacher on #2

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}\sin x + x^{\frac{1}{2}}\cos x$

$$\frac{d}{dx}\left[\sec x\right] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] =$$

$$\frac{0\cdot\cos x - 1\cdot(-\sin x)}{\cos^2 x} = +\frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}$$

$$= \sec x \tan x$$

Didn't have Chain Rule, yet.
But now we do.

3.7 W.
3.8 R
Test
Fri.

$$\frac{d}{dx}\left[\sec x\right] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{d}{dx}\left[(\cos x)^{-1}\right]$$

$$= -1(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x}$$