70

$$f(x) = \overset{h}{x}\,\overset{g}{g(x^2)}$$

$$f'g + fg'$$

$$(hg)' = h'g + hg'$$

$$f'(x) = 1 \cdot g(x^2) + x g'(x^2) \cdot 2x$$

$$= g(x^2) + 2x^2 g'(x^2)$$

$$f''(x) = 2x g'(x^2) + 4x g'(x^2) + 2x^2 g''(x^2) \cdot 2x$$

$$= 2x g'(x^2) + 4x g'(x^2) + 4x^3 g''(x^2)$$

49. The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse," that is, an ellipse whose axes are not parallel to the coordinate axes. Find the points at which this ellipse crosses the x-axis and show that the tangent lines at these points are parallel. $\to y=0$

$eqn2 := x^2 - x \cdot y + y^2 = 3$

$$x^2 - x\,y + y^2 = 3$$

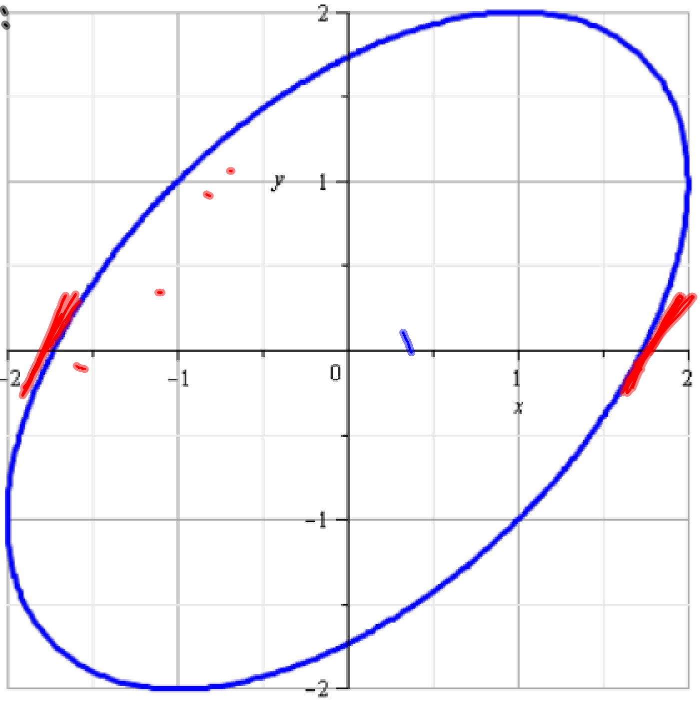
$implicitplot(eqn2, x = -10..10, y = -10..10, color = blue, thickness = 3, numpoints = 100000, gridlines = true)$

x-intercepts: $y=0$:

$x^2 = 3$

$\boxed{x = \pm\sqrt{3}}$

Diff:



$2x - 1\cdot y - xy' + 2yy' = 0$

$-xy' + 2yy' = y - 2x$

$y'(-x+2y) = y - 2x$

$\boxed{y' = \dfrac{2x-y}{x-2y}}$

$y'\Big|_{\substack{x=\sqrt{3}\\ y=0}} = \dfrac{2\sqrt{3}}{\sqrt{3}} = 2 = m_{tan}$

$y'\Big|_{\substack{x=-\sqrt{3}\\ y=0}} = \dfrac{2(-\sqrt{3})}{-\sqrt{3}} = 2 = m_{tan}$

So, they're parallel.

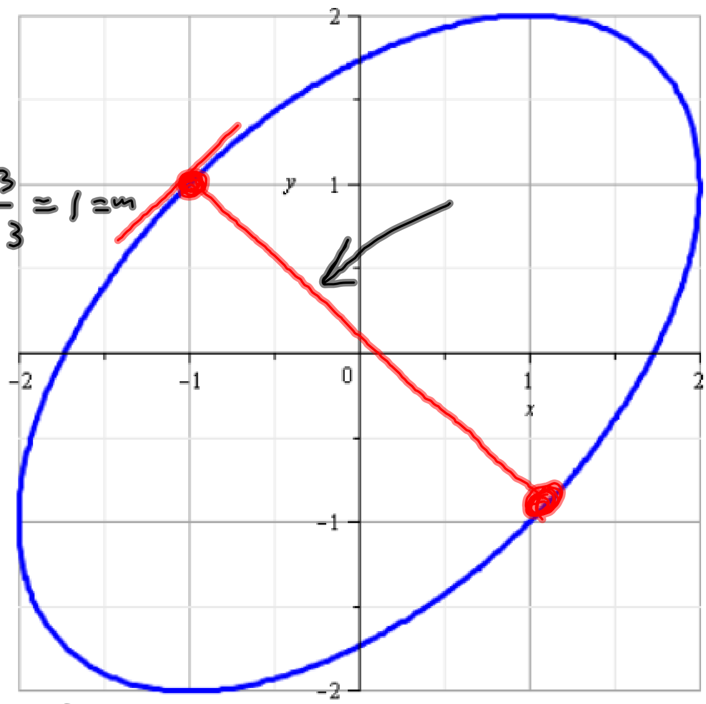
50. (a) Where does the normal line to the ellipse $x^2 - xy + y^2 = 3$ at the point $(-1, 1)$ intersect the ellipse a second time?

(b) Illustrate part (a) by graphing the ellipse and the normal line.

$eqn2 := x^2 - x \cdot y + y^2 = 3$

$$x^2 - xy + y^2 = 3$$

implicitplot(*eqn2*, $x = -10..10$, $y = -10..10$, *color* = *blue*, *thickness* = 3, *numpoints* = 100000, *gridlines* = *true*)



$\boxed{y' = \frac{2x - y}{x - 2y}}$

$y'\Big|_{(-1,1)} = \frac{2(-1) - 1}{-1 - 2(1)} = \frac{-3}{-3} = 1 = m$

$m_{tan} = 1$

$m_{normal} =$

$m_{\perp} = -1$

$y = m(x - x_1) + y_1$

$= -(x + 1) + 1$

$\boxed{y = -x}$

Substitution method.

$y = -x$ & $x^2 - xy + y^2 = 3$

$x^2 - x(-x) + (-x)^2 = 3$

$x^2 - xy + y^2 = 3$

$x^2 - x(-x) + (-x)^2 = 3$

$x^2 + x^2 + x^2 = 3$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$y = \mp 1$

$(1, -1)$ & $(-1, +1)$

$y = 2x - 3$

$x + y = 7$

$x + (2x - 3) = 7$

3.5 #88

$$|x| = \sqrt{x^2}$$

$$\frac{d}{dx}\left[|x|\right] = \frac{x}{|x|}$$

$$\frac{d}{dx}\left[|x|\right] = \frac{d}{dx}\left[\sqrt{x^2}\right] = \frac{d}{dx}\left[(x^2)^{\frac{1}{2}}\right]$$

$$= \frac{1}{2}(x^2)^{\frac{1}{2}-1}\cdot(2x) = x\cdot(x^2)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$$

$$\frac{d}{dx}\left[|\sin x|\right] = ?$$

$$|\sin x| = \sqrt{(\sin x)^2} = (\sin^2 x)^{\frac{1}{2}}$$

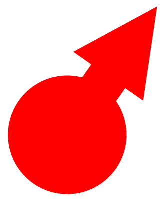
$$\frac{d}{dx}\left[|\sin x|\right] = \frac{d}{dx}\left[(\sin^2 x)^{\frac{1}{2}}\right]$$

$$= \frac{1}{\cancel{2}}(\sin^2 x)^{-\frac{1}{2}}(\cancel{2}\sin x)\cos x = \frac{\sin x \cos x}{\sqrt{\sin^2 x}}$$

$$= \frac{\sin x \cos x}{|\sin x|}$$

$$\frac{d}{dx}\left[f(g(h(x)))\right]$$

$$= f'(g(h(x)))\cdot g'(h(x))\cdot h'(x)$$



$$f(x) = x^{\frac{1}{2}} \qquad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

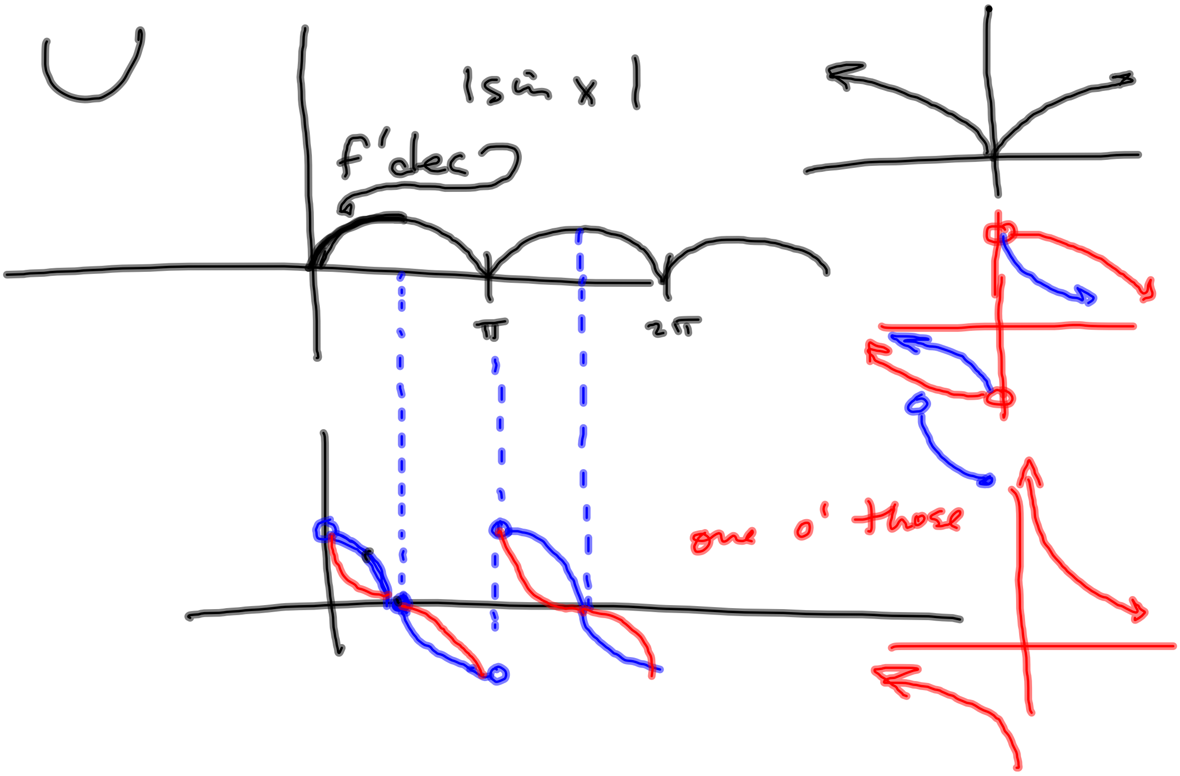
$$f'(\sin^2 x) = \frac{1}{2}(\sin^2 x)^{-\frac{1}{2}}$$

$$g(x) = x^2 \qquad g'(x) = 2x$$

$$g'(\sin x) = 2\sin x$$

$$h'(x) = \cos x$$

$$h(x) = \sin x$$



$$x^{-\frac{3}{2}} = \frac{1}{\sqrt{x^3}} = \frac{1}{(\sqrt{x})^3} = \frac{1}{x\sqrt{x}}$$

$$(\sqrt{x})^2 = \sqrt{x}\,\sqrt{x} = x$$

$$\sqrt{x^2} = |x|$$

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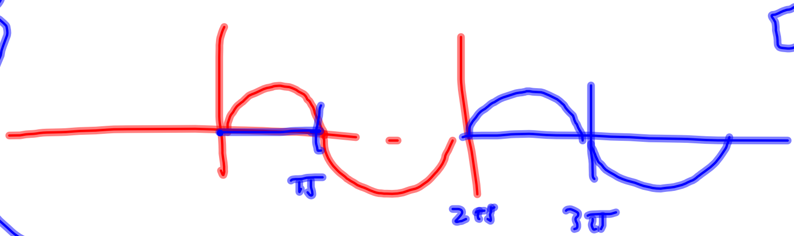
$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \ge 0 \\ -\sin x & \text{if } \sin x < 0 \end{cases}$$

$x \in$ ow!

waste of Time.



$[0, \pi], [2\pi, 3\pi], \ldots$

$[2n\pi, (2n+1)\pi]$

$n = 0, 1, 2, \ldots$

Painful

$x^{\frac{1}{2}-(-\frac{1}{2})}$
$= x^{1}$

$x^{\frac{1}{2}} + x^{-\frac{1}{2}} = x^{-\frac{1}{2}}\left[\frac{x^{\frac{1}{2}}}{x^{-\frac{1}{2}}} + \frac{x^{-\frac{1}{2}}}{x^{-\frac{1}{2}}}\right]$

$= x^{-\frac{1}{2}}\left[x + 1\right]$

$\sqrt{r^2+1} - r^2 (r^2+1)^{-\frac{1}{2}}$

$= (r^2+1)^{\frac{1}{2}} - r^2 (r^2+1)^{-\frac{1}{2}}$

$x^{-\frac{1}{2}-(-\frac{1}{2})}$
$= x^{0} = 1$

$= (r^2+1)^{-\frac{1}{2}}\left[\frac{(r^2+1)^{\frac{1}{2}}}{(r^2+1)^{-\frac{1}{2}}} - r^2\right]$

$= (r^2+1)^{-\frac{1}{2}}\left[r^2+1 - r^2\right]$

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