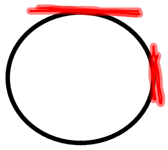
3.6 II Please hand in #s 49, 50. #50 requires some sort of grapher capability, either with a hand-held, or with a free grapher online.

3.6 IMPLICIT DIFFERENTIATION

Sometimes an equation gives a relationship between x and y that does NOT give y as a function of x. Consider the circle of radius 5, centered at the origin:

$$x^2 + y^2 = 25$$



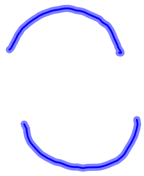
When we solve for y, we get *two* expressions (A plus and a minus version).

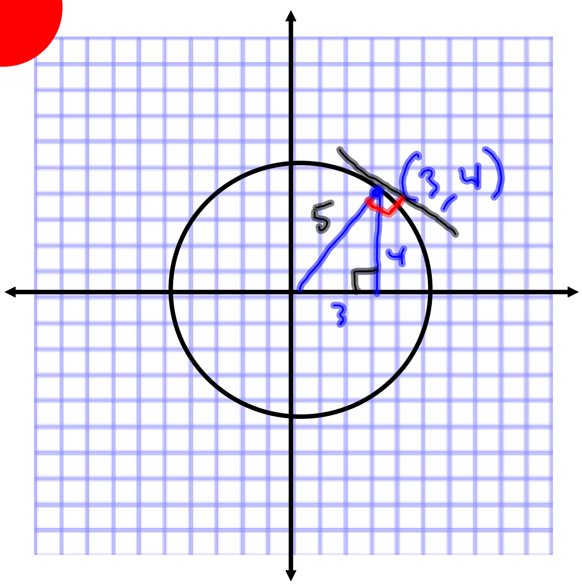
$|y| = \sqrt{y^2} = \sqrt{25-x^2}$

$y = \pm\sqrt{25-x^2}$

$y = \sqrt{25-x^2}$

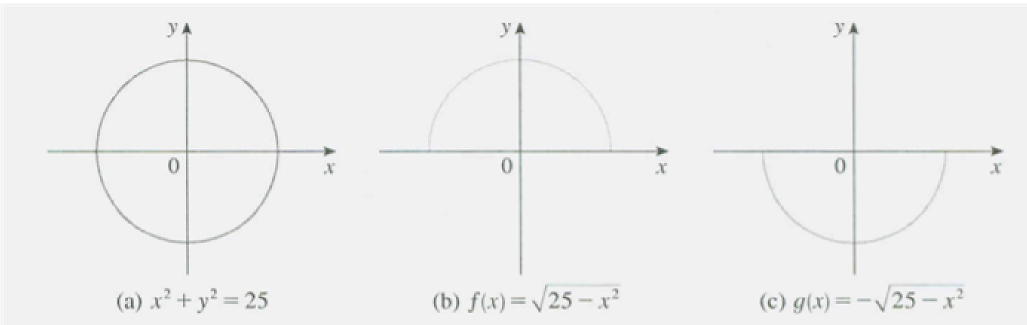
$y = -\sqrt{25-x^2}$





The question: what's the slope @ (3, 4)?

$m = -\frac{3}{4}$, by analytic geometry. Now, be quiet.



(a) $x^2 + y^2 = 25$ (b) $f(x) = \sqrt{25 - x^2}$ (c) $g(x) = -\sqrt{25 - x^2}$

Even though y is not strictly (**explicitly**) a function of x, we *can* say some things about the rate of change in y with respect to x.

Using the Chain Rule, and viewing y as an **implicit** function of x, we can still differentiate and find an expression for $y' = dy/dx$.

Feb 16-9:21 AM

So let's do it explicitly:

$$y = \sqrt{25-x^2} = (25-x^2)^{\frac{1}{2}}$$

$$\Longrightarrow y' = \frac{1}{\cancel{2}}(25-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

Planting Seed.

$$= \frac{-x}{\sqrt{25-x^2}}$$

NOTICE THIS THE SAME AS ... $\frac{-x}{y}$, since $y=\sqrt{25-x^2}$?

$$\text{So, } y'\Big|_{\substack{x=3\\y=4}} = \frac{-3}{\sqrt{25-(3)^2}} = \frac{-3}{\sqrt{16}} = \boxed{-\frac{3}{4} = m}$$

This implicit differentiation technique allows us to say a *lot* about curves, even when it's ***impossible to solve for y explicitly***.

(a) If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.

(b) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, 4).

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2)\frac{dy}{dx} = 2y\frac{dy}{dx} = 2yy'$$

$$\frac{d}{dx}\left[f(x)^2\right] = 2f(x)\cdot f'(x)$$

General Power Rule

Differentiate both sides of $x^2+y^2=25$, with respect to x:

$$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}[25]$$

$$2x + 2yy' = 0$$

Solve for y'

$$2yy' = -2x$$

$$y' = \frac{-2x}{2y}$$

$$y' = -\frac{x}{y}$$

Find the slope of the tangent to circle @ (3,4)

$$y'\Big|_{\substack{x=3\\y=4}} = \boxed{-\frac{3}{4} = m}$$

So an eq'n of the tangent @ (3,4) is

$$y = m(x - x_1) + y_1$$

$$y = -\frac{3}{4}(x-3) + 4$$

1–4

(a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x. No!

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a). No!

$$(fg)' = f'g + fg'$$

$$2x \cdot y + x^2 \cdot y'$$

8. $2x^3 + \underbrace{x^2}_{f}\underbrace{y}_{g} - xy^3 = 2$

$$6x^2 + 2xy + x^2y' - y^3 - x \cdot 3y^2y' = 0$$

$$x^2y' - 3xy^2y' = y^3 - 6x^2 - 2xy$$

$$y'(x^2 - 3xy^2) = y^3 - 6x^2 - 2xy$$

$$\boxed{y' = \frac{y^3 - 6x^2 - 2xy}{x^2 - 3xy^2}}$$

$2y^3+y^2-y^5=0$

$-y^2(y^3-2y+1)=0$

3

38. (a) The curve with equation

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

$1 \quad -2 \quad +1 \quad =0$

has been likened to a bouncing wagon. Use a computer algebra system to graph this curve and discover why.

(b) At how many points does this curve have horizontal tangent lines? Find the x-coordinates of these points.

Looks like 9

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with(plots) :
eqn1 := 2·y^3 + y^2 − y^5 = x^4 − 2·x^3 + x^2
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$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

```
implicitplot(eqn1, x = −10 ..10, y = −10 ..10, color = blue, thickness = 3, numpoints = 100000, gridlines = true)
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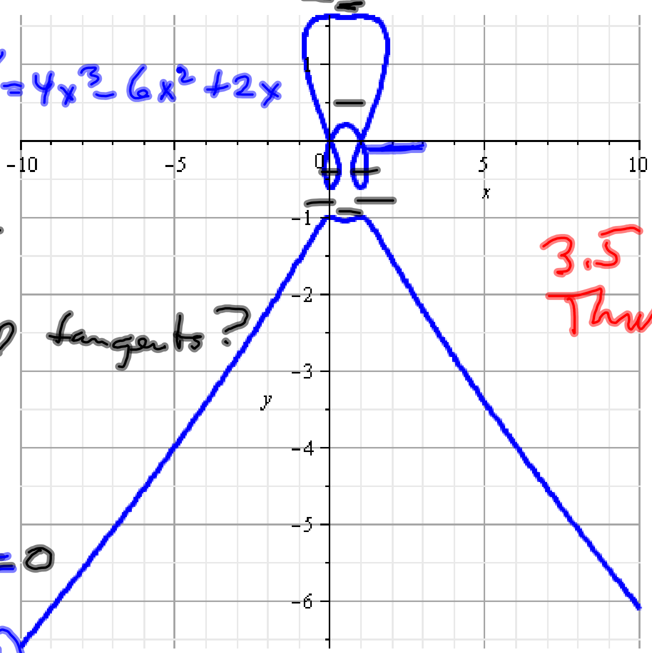
Differentiate:

$6y^2y' + 2yy' - 5y^4y' = 4x^3 - 6x^2 + 2x$



$$y' = \frac{4x^3-6x^2+2x}{6y^2+2y-5y^4}$$

Where are horizontal tangents?

$y'=0 \implies$

$4x^3-6x^2+2x=0$

$\implies 2x(2x^2-3x+1)=0$

$\implies 2x(2x-1)(x-1)$

$\implies x=0, \frac{1}{2}, 1$

3.5 Thursday.