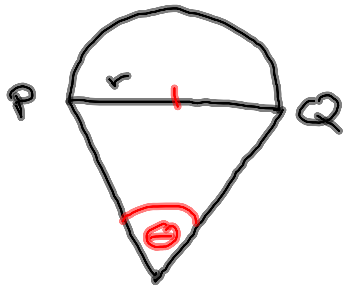
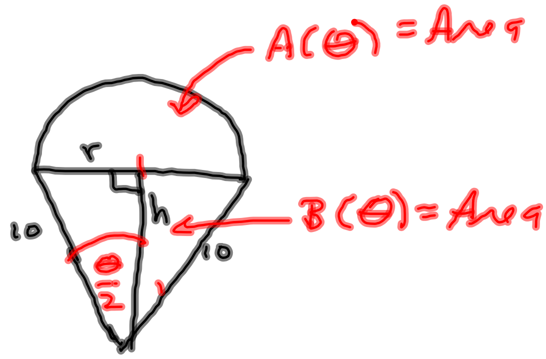
§3.3 II due now:

3.4 #50





θ is shrinking

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}$$

$A(\theta) = \pi r^2(\theta) \cdot \frac{1}{2}$

Need r as func. of θ.

$\frac{r}{10} = \sin\left(\frac{\theta}{2}\right)$

$r = 10 \sin\left(\frac{\theta}{2}\right)$

$A(\theta) = \frac{\pi}{2} \cdot \left(10 \sin\left(\frac{\theta}{2}\right)\right)^2$

$= 5\pi \left(\sin\left(\frac{\theta}{2}\right)\right)^2 \rightarrow 50$

$A(\theta) = \frac{\pi}{2} r^2$

$B(\theta) = \frac{1}{2} b \cdot h$

$= \frac{1}{2} r \cdot h$

$= \frac{1}{2} r \cdot 10 \cos\left(\frac{\theta}{2}\right)$

$= 5 \left(10 \sin\left(\frac{\theta}{2}\right)\right)\left(\cos\left(\frac{\theta}{2}\right)\right) \rightarrow 50$

$$\frac{A}{B} = \frac{5\pi\left(\sin\left(\frac{\theta}{2}\right)\right)^2}{50 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)} = \frac{\pi \sin\left(\frac{\theta}{2}\right)}{\cancel{10} \cos(\theta/2)}$$

$$= \frac{\pi \tan\left(\frac{\theta}{2}\right)}{\cancel{10}} \xrightarrow{\theta \to 0} 0$$

Feb 14-12:21 PM

[4] **THE POWER RULE COMBINED WITH THE CHAIN RULE** If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

General Power Rule

Alternatively, $$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

Same Deal.

$$\frac{d}{dx}\left[\sin^{72} x\right] = \underline{\left(72 \sin^{71} x\right)}\underline{\left(\cos x\right)}$$

$$(\sin x)^{72}$$

$$\frac{d}{dx}\left[\left(x^2-3x\right)^{72}\right] = 72\left(x^2-3x\right)^{71}(2x-3)$$

51–54 Find an equation of the tangent line to the curve at the given point.

General Power Rule

52. $y = \sin x + \sin^2 x$, (0, 0)

$y = m(x - x_1) + y_1$

$y = 1(x-0)+0$

$y = x$

$y'\Big|_{x=0} = \left(\cos x + 2\sin x \cdot \cos x\right)\Big|_{x=0} = 1$

$$\boxed{\frac{d}{dx}\left[(\sin x)^2\right] = 2(\sin x)^1 \cdot \cos x}$$

$$\frac{d}{dx}\left[(\sin x)(\sin x)\right] = \cos x \cdot \sin x + \sin x \cdot \cos x = 2\sin x\cos x$$

60. Find the x-coordinates of all points on the curve $y = \sin(2x) - 2\sin x$ at which the tangent line is horizontal.

$y' = 2\cos(2x) - 2\cos x \overset{\text{SET}}{=} 0$

$\Longrightarrow \cos(2x) - \cos x = 0$

$\left(\cos^2 x - \sin^2 x = \cos^2 - (1 - \cos^2 x) = 2\cos^2 x - 1 \right.$

Scratch

$\left. \cos(x+y) = \cos x \cos y - \sin x \sin y \right)$

$\Longrightarrow 2\cos^2 x - \cos x - 1 = 0$

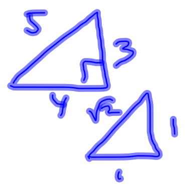
$2u^2 - u - 1 = 0$

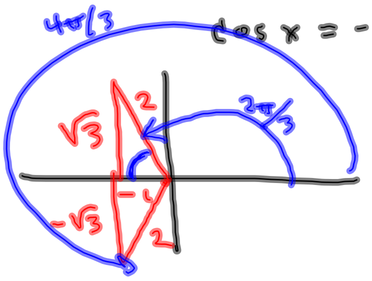
$(2u+1)(u-1) = 0$

$u = -\frac{1}{2}$ OR $u = 1$

$\cos x = -\frac{1}{2}$ OR $\cos x = 1$

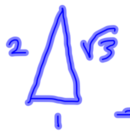
$x = 0, \pm 2\pi, \pm 4\pi, \ldots$





$\pi - \frac{\pi}{3} = \frac{2\pi}{3} \pm 2n\pi$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3} \pm 2n\pi$



$fg' + gf'$ $\qquad$ $f'g + fg'$

$$\frac{gf' - fg'}{g^2} \qquad \frac{f'g - fg'}{g^2}$$

68. Suppose f is differentiable on $\mathbb{R}$ and α is a real number. Let $F(x) = f(x^\alpha)$ and $G(x) = [f(x)]^\alpha$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

$F'(x) = f'(x^\alpha) \cdot \alpha x^{\alpha-1}$

$G'(x) = \alpha f(x)^{\alpha-1} \cdot f'(x)$

72. If $F(x) = f(xf(xf(x)))$, where $f(1) = 2, f(2) = 3, f'(1) = 4, f'(2) = 5$, and $f'(3) = 6$, find $F'(1)$.