3.3 questions?

3.4 Questions? 46

$$\lim_{x\to 0} \frac{\sin(x^2)}{x} = \lim_{x\to 0}\left(\frac{x}{x}\cdot\frac{\sin(x^2)}{x}\right) = \lim_{x\to 0}\left(x\cdot\frac{\sin(x^2)}{x^2}\right)$$

$$= \lim_{x\to 0} x \cdot \boxed{\lim_{x\to 0}\frac{\sin(x^2)}{x^2}} = 0$$

$$\lim_{x^2\to 0}\left(\frac{\sin(x^2)}{x^2}\right) = 1$$

Another way of saying

$f(2)$ is $f(x)\Big|_{x=2}$ "$f(x)$ evaluated @ $x=2$"

3.3 #66:

Want the derivative of $\frac{h(x)}{x}$, evaluated at $x=2$

$\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{h'(x)\,x - h(x)}{x^2}$ $h'(2)=-3$ $h(2)=4$

$$\therefore \frac{d}{dx}\left[\frac{h(x)}{x}\right]\Bigg|_{x=2} = \frac{h'(2)\cdot 2 - h(2)}{2^2} = \frac{(-3)(2)-4}{4} = \frac{-6-4}{4} = -\frac{5}{2}$$

#69: g is dif ble. Find the derivative:

(a) $y = xg(x) \Longrightarrow y' = 1\cdot g(x) + x\cdot g'(x)$
$= g(x) + xg'(x)$

(b) $y = \frac{x}{g(x)}$ I'd rather do $y = \frac{x^2}{g(x)} \Longrightarrow$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{gf' - fg'}{g^2}$$

$$y' = \frac{2x\cdot g(x) - x^2\cdot g'(x)}{(g(x))^2}$$

3.4 #40

$$\lim_{x\to 0} \frac{\sin(4x)}{\sin(6x)} = \lim_{x\to 0}\left(\frac{\sin(4x)}{4x}\cdot\frac{6x}{\sin(6x)}\cdot\frac{4}{6}\right)$$

$$= \frac{4}{6}\lim_{x\to 0}\left(\frac{\sin(4x)}{4x}\right)\left(\lim_{x\to 0}\frac{6x}{\sin(6x)}\right)$$

$$= \frac{4}{6} = \boxed{\frac{2}{3}}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

$$\cancel{\lim_{x\to 0}\frac{\sin(4x)}{\sin(6x)} = \frac{4\sin x}{6\sin x} = \frac{4}{6} = \frac{2}{3}}$$

No!

3.5 THE CHAIN RULE

A big part of the skill-set involved in the chain rule is recognizing composite functions and understanding the notation.

1–6 Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

§3.4 Tuesday

1. $y = \sin(4x)$

$f(g(x))$

$f(x) = \sin x$

$g(x) = 4x$

$F(x) = (4x - x^2)^{100}$ $= f(g(x))$

$x^{100} = f(x)$

$4x - x^2 = g(x)$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx}$$

Feb 10-3:36 PM

THE CHAIN RULE If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

"Derivative of the outer function with respect to the inner function times the derivative of the inner function respect to x."

In practice, I think of this as working my way in from the outside.

Another way of thinking of it is

"The rate of change of a function that contains another function is...

... how fast the outer function changes with respect to the inner function, times how fast the inner function is changing with respect to time."

Discussion of the Chain Rule thru Visual Calculus Website:

http://archives.math.utk.edu/visual.calculus/2/chain_rule.4/index.html

$\boxed{4}$ **THE POWER RULE COMBINED WITH THE CHAIN RULE** If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively, $$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

Feb 11-10:43 AM

A pebble is dropped into a pond.

The ripples are expanding outward at a rate of 30 feet per second.

How fast is the area inside the ripples growing, when $t = 3$ seconds?

We answer this question in two ways:

1. Direct calculation.

$A = \pi r^2$

$A(t) = \pi(30t)^2$

$= 900\pi t^2$

$\left.\frac{dA}{dt}\right|_{t=3} = \left.1800\pi t\right|_{t=3} = 1800\pi(3)$

$= 5400\pi \frac{ft^2}{s}$

$r = 30t$

2. Chain Rule

$A = \pi r^2$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$= 2\pi r \cdot 30$

what's $r(3) =$
$30 \cdot 3 = 90$

So

$2\pi r \cdot 30 =$

$2\pi(90)(30)$

$= 5400\pi \frac{ft^2}{sec}$

Better yet

$A(r(t)) = \pi(r(t))^2 = \pi(\underbrace{30t}_{r(t)})^2$

$A(r) = \pi r^2$

$\frac{dA}{dt} = \underbrace{2\pi(30t)'}_{\frac{dA}{dr}} \cdot \underbrace{30}_{\frac{dr}{dt}}$

There doesn't seem to be much difference between the two methods, but what about THIS one?

$h(x) = (3x^2 - 5x)^2 = 9x^4 - 30x^3 + 25x^2$

$h'(x) = 36x^3 - 90x^2 + 50x$ w/o chain

$h'(x) = \underbrace{2(3x^2-5x)^1}_{\frac{df}{dg}}\underbrace{(6x-5)}_{\frac{dg}{dx}} = 2(18x^3 - 15x^2 - 30x^2 + 25x)$

$= 36x^3 - 90x^2 + 50x$

$f(x) = x^2$

$g(x) = 3x^2 - 5x$

$h(x) = (3x^2 - 5x)^{17}$

$h'(x) = 17(3x^2 - 5x)^{16}(6x-5)$

Feb 10-3:34 PM

7–46 Find the derivative of the function.

12. $f(t) = \sqrt[3]{1 + \tan t}$

24. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$

$$28.\ y = \frac{\cos \pi x}{\sin \pi x + \cos \pi x}$$

It's better to enclose arguments in parentheses, when any ambiguity is possible.

$$28.\ y = \frac{\cos(\pi x)}{\sin(\pi x) + \cos(\pi x)}$$

Feb 10-3:53 PM

51–54 Find an equation of the tangent line to the curve at the given point.

52. $y = \sin x + \sin^2 x$, $(0, 0)$

60. Find the x-coordinates of all points on the curve $y = \sin 2x - 2\sin x$ at which the tangent line is horizontal.

68. Suppose f is differentiable on $\mathbb{R}$ and α is a real number. Let $F(x) = f(x^\alpha)$ and $G(x) = [f(x)]^\alpha$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$.

72. If $F(x) = f(xf(xf(x)))$, where $f(1) = 2, f(2) = 3, f'(1) = 4$, $f'(2) = 5$, and $f'(3) = 6$, find $F'(1)$.

Feb 11-11:29 AM