85

§2.3

$f(x) = ax^3 + bx^2 + cx + d$

f has horizontal tangents @ $(2,0)$ & $(-2,6)$

Find a, b, c, d

$f'(x) = 3ax^2 + 2bx + c$

✓ $f'(2) = 12a + 4b + c = 0$

✓ $f'(-2) = 12a - 4b + c = 0$

✓ $f(2) = 8a + 4b + 2c + d = 0$

✓ $f(-2) = -8a + 4b - 2c + d = -6$

4 equations
4 unknowns

No. +6, Steve.

$$\left[\begin{array}{cccc|c} 12 & 4 & 1 & 0 & 0 \\ 12 & -4 & 1 & 0 & 0 \\ 8 & 4 & 2 & 1 & 0 \\ -8 & 4 & -2 & 1 & -6 \end{array}\right]$$

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… 0 1 0 2.25   ]
… 0 0 1 -3    ]]
Ans▸Frac
…1 0 0 0 -3/16]
…0 1 0 0 0    ]
…0 0 1 0 9/4  ]
…0 0 0 1 -3   ]]
```

$a = -\frac{3}{16}$

$b = 0$

$c = \frac{9}{4}$

$d = -3$

$f(x) = -\frac{3}{16}x^3 + \frac{9}{4}x - 3$

$f(2) = -\frac{3}{16}(2^3) + \frac{9}{4}(2) - 3$

$= -\frac{3}{2} + \frac{9}{2} - 3 = \frac{6}{2} - 3 = 0$

$f(-2) = -\frac{3}{16}(-2)^3 + \frac{9}{4}(-2) - 3$

$= \frac{3}{2} - \frac{9}{2} - 3 = -3 - 3 = -6$

$f'(x) = -\frac{9}{16}x^2 + \frac{9}{4}$

$f'(2) = -\frac{9}{16} \cdot 4 + \frac{9}{4} = 0$

$f(-2) = -\frac{9}{16} \cdot 4 + \frac{9}{4} = 0$

Sweet. Now do it for $(2,6)$, fool. Idiot.

#99

$$\lim_{x \to 1} \frac{x^{1000}-1}{x-1} = 1000 \text{ is } f'(a) \text{ for}$$

$$f(x) = x^{1000}$$

$$a = 1$$

$$f(x) = x^{1000}$$

$$f'(x) = 1000x^{999}$$

$$\boxed{f'(1) = 1000}$$

$$(x^{500}-1)(x^{500}+1)$$

$x^{1000} - 1 = 0$ when $x = 1$.

So $x-1$ is a factor.

$$\begin{array}{r|cccccccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -1 \\
 & & 1 & 1 & 1 & 1 & & & & & & & & 1 \\
\hline
 & 1 & 1 & 1 & 1 & 1 & \cdots & & & & & & 1 & 0 \\
 & x^{999} & \cdots & & & & & & & & x^2 & x^1 & c & r
\end{array}$$

So $x^{1000} - 1 = (x-1)(x^{999} + x^{998} + x^{997} + \cdots + x^1 + 1)$

$$\therefore \lim_{x \to 1} \frac{x^{1000}-1}{x-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^{999} + \cdots + 1)}{\cancel{x-1}}$$

$$= \lim_{x \to 1} (x^{999} + x^{998} + \cdots + 1x^0) = 1000$$

$$x^6 - 1 = (x-1)(x^5 + x^4 + x^3 + x^2 + x + 1)$$