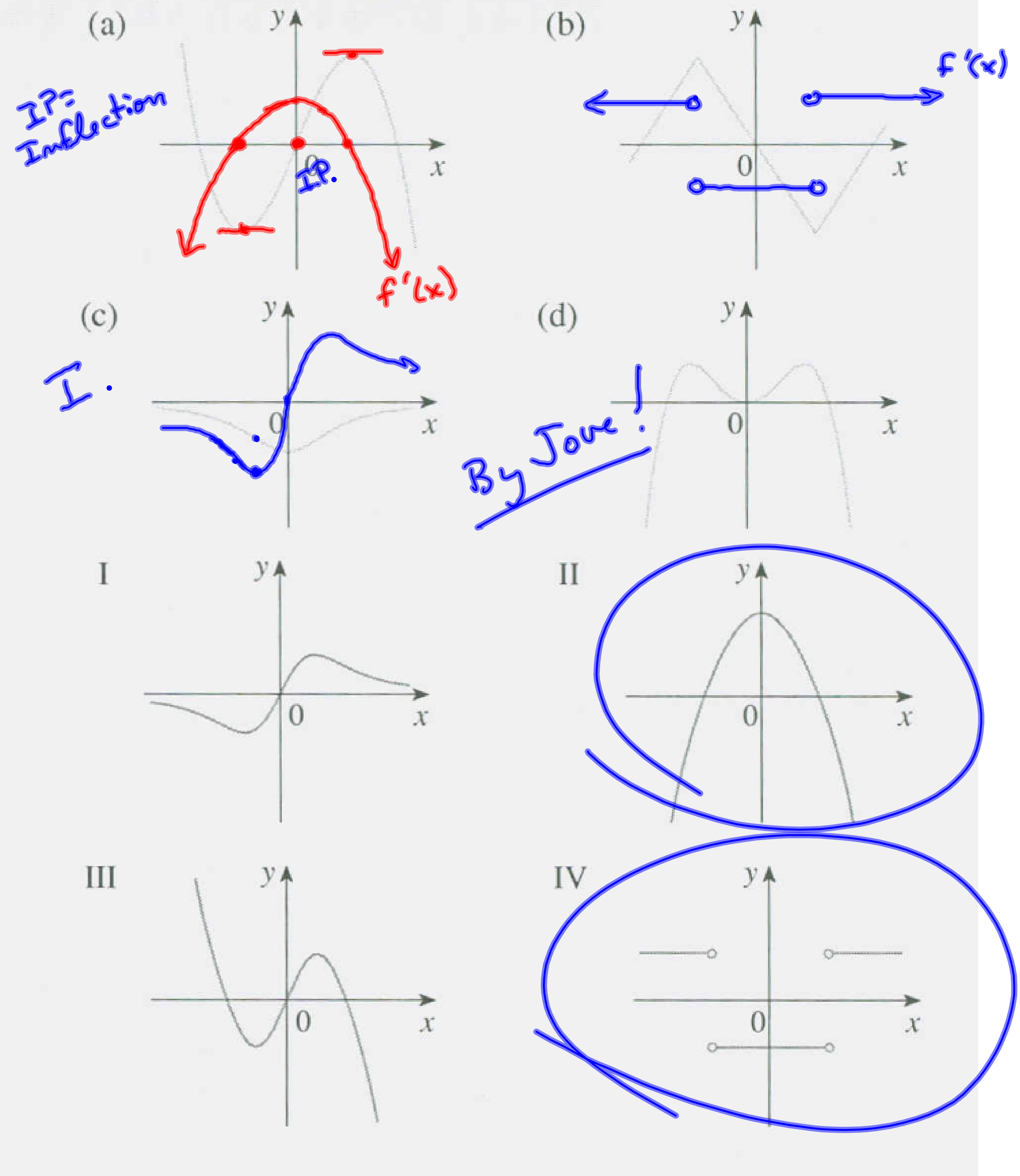
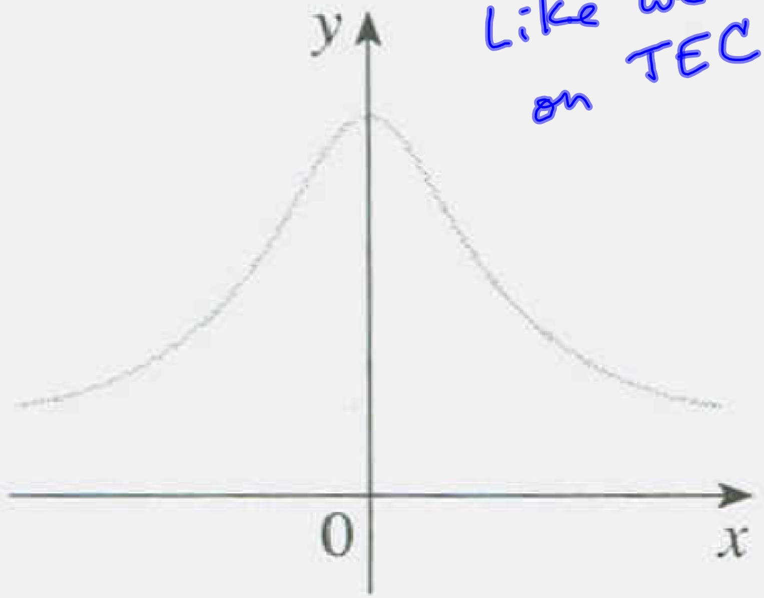
3. Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.



4–11 Trace or copy the graph of the given function f. (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of f' below it.

6.



Like we saw on TEC.

These are no different from what we did in 3.1, except we're using x instead of a, and we're thinking of the result as a function of x. The mechanics are exactly the same as before. Also we will tend to use this formulation,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

rather than this formulation:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Recall, from 3.1 that when working with polynomials, this second formulation required you to split off the factor of $x - a$ and cancel it, whereas in the first formulation, you simply expand the numerator, and factor out h.

In practice, I typically just work on the difference quotient and save the limit until the last step. *It's important not to connect limit with non-limit by equals operator! If you're sloppy about this, I WILL deduct points.*

$$\frac{f(x+h) - f(x)}{h} = \text{... simplify and then pass to the limit ...} \xrightarrow{h \to 0}$$

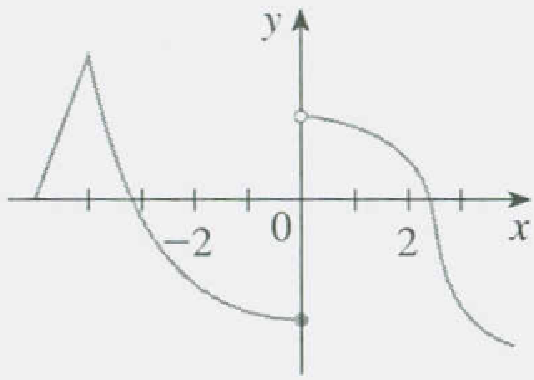
17–27 Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

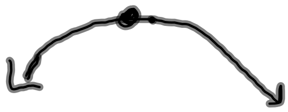
21. $f(x) = x^3 - 3x + 5$

Recall, we did a cubic in class last time using the first formulation. This time we use the "h".

33–36 The graph of f is given. State, with reasons, the numbers at which f is not differentiable.

33.





$x = -4$ $\quad \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} \neq \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h}$

Positive $\neq$ Negative

$x = 0$ $\quad$ f ain't cont.

How might a function fail to have a derivative at some $x = a$?

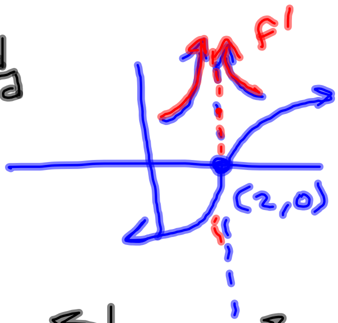
Not continuous at a.
Not "smooth" at a.
Absolute value comes to a "point."

Functions like $(x-a)^{2/3}$ actually "go vertical" at $x = a$.

In general, any power-type function, $(x-a)^b$ where the power, b, is between 0 and 1 is going to be continuous everywhere, but will have a vertical tangent at $x = a$.

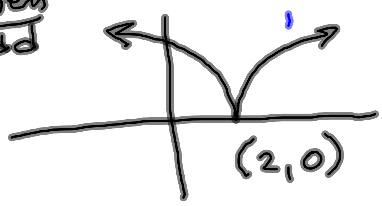
Finally, read Definition 3.

$(x-2)^{\frac{1}{3}} = (x-2)^{\frac{\text{odd}}{\text{odd}}}$



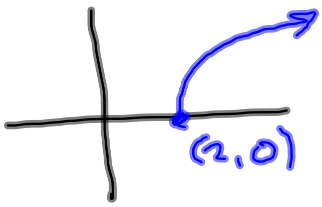
$f'(2) \nexists$

$(x-2)^{\frac{2}{3}} = (x-2)^{\frac{\text{even}}{\text{odd}}}$



$f'(2) \nexists$

$(x-2)^{\frac{1}{2}}$



$$f(x) = 2.7x^2 - 13.5x + 11.113$$
$$= ax^2 + bx + c$$

Find $f'(x)$.

Let $a = 2.7$, $b = -13.5$, $c = 11.113$

$$\frac{f(x+h) - f(x)}{h} = \frac{a(x+h)^2 + b(x+h) + c - (ax^2 + bx + c)}{h}$$

$$= \frac{a(x^2 + 2xh + h^2) + bx + bh + c - ax^2 - bx - c}{h}$$

$$= \frac{ax^2 + 2axh + ah^2 + bh - ax^2}{h}$$

$$= \frac{h(2ax + ah + b)}{h} = 2ax + ah + b \quad (h \neq 0)$$

$$\xrightarrow{h \to 0} 2ax + b = f'(x)$$

$$\boxed{= 5.4x - 13.5 = f'(x)}$$

\# 44, §3.1

$$100000\left(1-\frac{t}{60}\right)^2 = 100000\left(1-\frac{t}{30}+\frac{t^2}{3600}\right)$$

$$= 100{,}000 - \frac{10000}{3}t + \frac{250}{9}t^2$$

$$= c \quad + bt \quad + at^2$$

$= at^2 + bt + c$, where

$$a = \frac{250}{9},\ b = -\frac{10000}{3},\ c = 100{,}000$$

Find $f'(t)$ (See previous page)