#14 §3.1 Questions?

$H(t) = +10t - 1.86t^2$

$H(t) = \frac{1}{2} g t^2 + v_0 t + H_0$

Initial height

gravity

initial velocity

(a) $H'(1)$

(b) $H'(a)$

$$\frac{H(a+h) - H(a)}{h} =$$

$$= \frac{10(a+h) - 1.86(a+h)^2 - (10a - 1.86a^2)}{h}$$

$$= \quad \dots \xrightarrow{h \to 0} \boxed{\text{FINAL ANS.}}$$

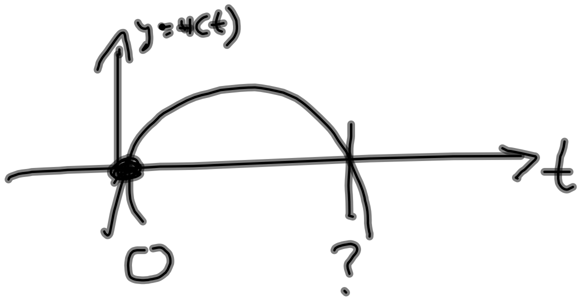
After (b), Let $a = 1$ & answer (a)

(c) $H(t) = 0$

$10t - 1.86t^2 = 0$

(d) $H'(t)$, where t is answer for c.

Feed answer for (c) into (b).



Like 3.1 #5

$y = \frac{x-1}{x-2} = f(x)$ Want m_{tan} @ $(3,2)$

$f(x) = \frac{x+5}{x+2}$ @ $(2, \frac{7}{4})$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{(x+h)+5}{(x+h)+2} - \frac{x+5}{x+2}}{h}$$

LCD = $(x+h+2)(x+2)$

$$= \frac{1}{h}\left[\left(\frac{x+h+5}{x+h+2}\right)\left(\frac{x+2}{x+2}\right) - \left(\frac{x+5}{x+2}\right)\left(\frac{x+h+2}{x+h+2}\right)\right]$$

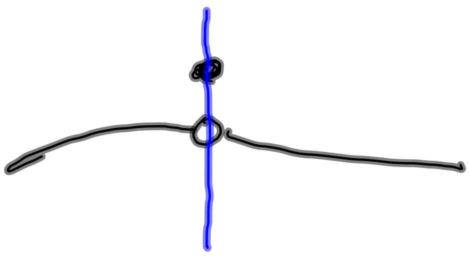
$$= \frac{1}{h}\left[\frac{x^2+2x+hx+2h+5x+10-(x^2+xh+2x+5x+5h+10)}{(x+h+2)(x+2)}\right]$$

$$= \frac{1}{h}\left[\frac{x^2+2x+hx+2h+5x+10-x^2-xh-2x-5x-5h-10}{(x+h+2)(x+2)}\right]$$

$$= \frac{1}{h}\left[\frac{hx+2h-xh-5h}{(x+h+2)(x+2}\right] = \frac{\cancel{h}}{\cancel{h}}\left[\frac{2-5}{LCD}\right] = \frac{-3}{(x+h+2)(x+2)}$$

$(h \neq 0)$

$$\xrightarrow{h \to 0} \frac{-3}{(x+2)(x+2)} = \frac{-3}{(x+2)^2}$$



C

$$\boxed{\lim_{x \to c} f(x) = f(c) \text{ when } f \text{ is cont}^{s}}$$

$$y = \frac{x+5}{x+2} \Longrightarrow y' = \frac{1(x+2) - (x+5)(1)}{(x+2)^2} = \frac{x+2-x-5}{(x+2)^2}$$

$$= -\frac{3}{(x+2)^2}$$

$$\frac{f'g - fg'}{g^2}$$

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$$\frac{f(x)-f(c)}{x-c} =$$

$$f(x) = \frac{x+5}{x+2}$$

$$LCD = (x+2)(c+2)$$

$$\frac{\frac{x+5}{x+2} - \frac{c+5}{c+2}}{x-c} = \frac{1}{x-c}\left[\left(\frac{x+5}{x+2}\right)\left(\frac{c+2}{c+2}\right) - \left(\frac{c+5}{c+2}\right)\left(\frac{x+2}{x+2}\right)\right]$$

$$= \frac{1}{x-c}\left[\frac{xc+2x+5c+10-(cx+2c+5x+10)}{(c+2)(x+2)}\right]$$

$$= \frac{1}{x-c}\left[\frac{xc+2x+5c+10-cx-2c-5x-10}{(c+2)(x+2)}\right]$$

$$= \frac{1}{x-c}\left[\frac{-3x+3c}{(c+2)(x+2)}\right] = \frac{1}{\cancel{x-c}}\left[\frac{-3\cancel{(x-c)}}{(c+2)(x+2)}\right]$$

$$= \frac{-3}{(c+2)(x+2)} \quad (x \neq c) \xrightarrow{x \to c} \frac{-3}{(c+2)(c+2)}$$

$$= -\frac{3}{(c+2)^2} = f'(c)$$

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{x \to c} \frac{f(x)-f(c)}{x-c} = \lim_{x \to c} \frac{f(c)-f(x)}{c-x}$$

$f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\frac{h}{h(\sqrt{x+h}+\sqrt{x})}=\frac{1}{\sqrt{x+h}+\sqrt{x}}\quad (h\neq 0)$$

$$\xrightarrow{h\to 0}\ \frac{1}{\sqrt{x}+\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}}=f'(x)}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

3.2 THE DERIVATIVE AS A FUNCTION

Higher-Order Derivatives

Recall: The derivative of f at a.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

New: The derivative as a function in and of itself. It's the function that reports the slope of f at any given value of x.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

It can be interpreted as the slope of the tangent line to f at x.

TEC Visual 3.2 shows an animation of Figure 2 for several functions.

http://www.stewartcalculus.com/tec/

$$f'(x) = y' = \underbrace{\frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}}_{\text{Liebniz Notation}} f(x) = Df(x) = D_x f(x)$$

3 DEFINITION A function f is **differentiable at** $\boldsymbol{a}$ if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

Fact: The domain of the derivative is always contained within the domain of the original function. Sometimes the derivative's domain is significantly smaller than the original function's domain.

Fact: (Store away for later) Continuity is necessary but not sufficient for existence of the derivative. We want to think of this in terms of "smoothness." If f is differentiable at a, then it's continuous at a. This is ~~given in the ...~~

4 THEOREM If f is differentiable at a, then f is continuous at a.

All functions that are differentiable at a are continuous at a. But some functions that are continuous at a are not differentiable at a.

Consider the absolute value function. It's discussed at length in Example 5. I usually don't do book examples in lecture, but I definitely want to say a few things about $f(x) = |x|$

Break it Down!

lim $= -1$

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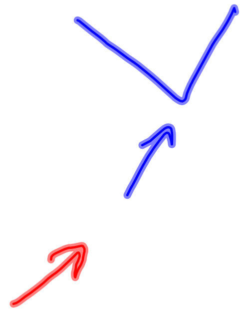
Smoothness.



$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

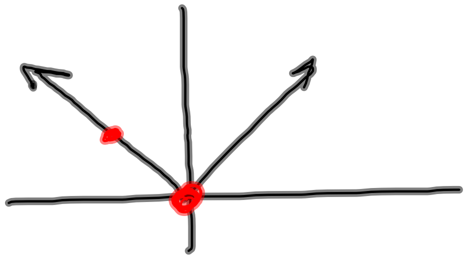
what's $f'(0)$?

$$f'(0) = m_{tan} \text{ @ } x=0 = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = ?$$

$h \to 0^-$

$\lim_{h\to 0^+}$ . . . . . $= ? = +1$



$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Really looking @

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} \quad \nexists$$

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-(0+h)-(-0)}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$$

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h-0}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$$|x^3-5x+2| = \begin{cases} x^3-5x+2 & \text{if } x^3-5x+2 \geq 0 \\ -(x^3-5x+2) & \text{if } x^3-5x+2 < 0 \end{cases}$$

If $f(x) = x^3 - x$, find and interpret $f''(x)$.

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$f'(x) = 3x^2 - 1$ See Example 2 for this one...

$$f''(x) = (f')'(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = \underline{\text{Liebniz}}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \frac{df}{dx} = \underline{\text{Velocity}}$$

$$f''(x) = \frac{d^2f}{dx^2} = \left(f'(x)\right)'$$

~~exraggerate~~

~~exaggerate~~

acceleration

$$= \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h}$$

Derivative of the derivative.

$f'''(x)$ $\underline{\text{Jerk}}$

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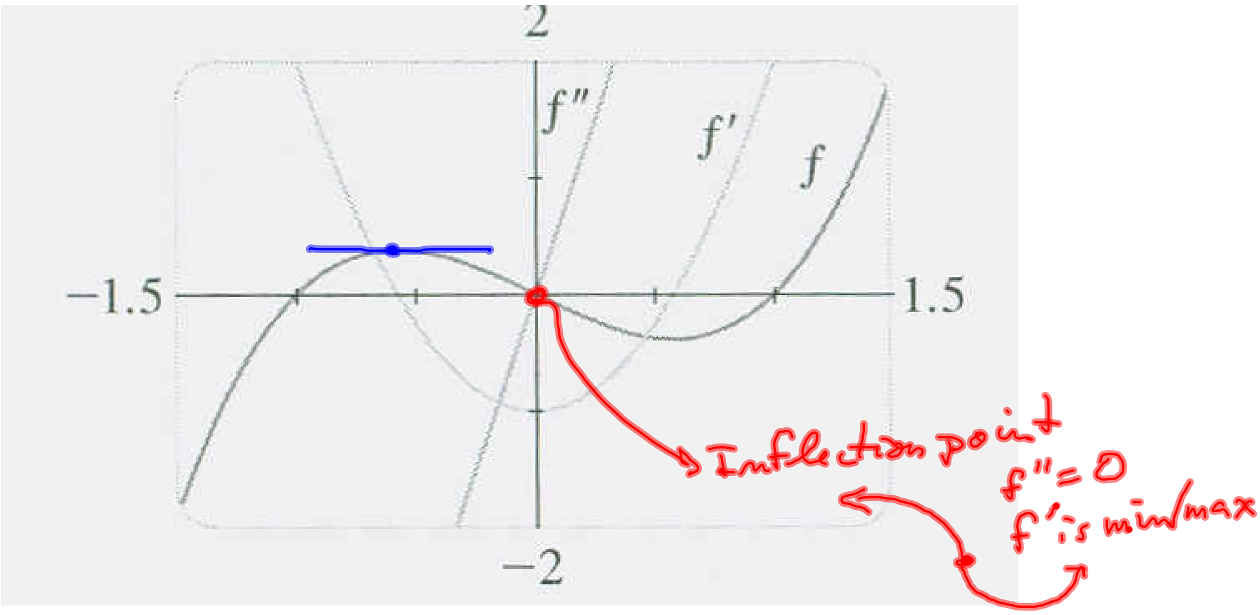
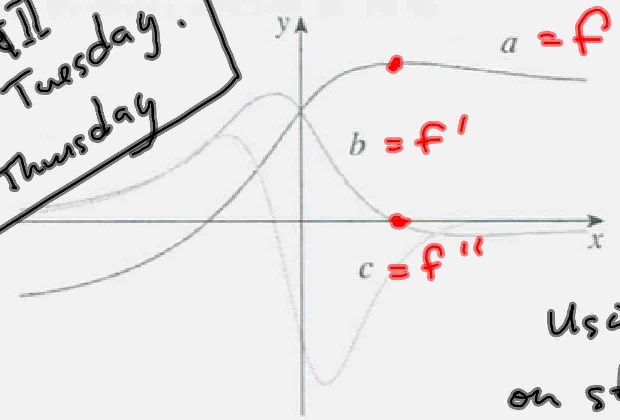


I kind of like the idea of a test question that asks *you* to determine, from an unlabeled graph of f, f', and f'', just which is which.

Feb 7-10:48 AM

39. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



§3.1 I & II due Tuesday.
3.2 Thursday

Using observations on steepness of f to look for f' & f''.

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Intuitively, it's handy to think of the second derivative as "acceleration". The rate of change of the rate of change of the function.

The third derivative is a bit tougher to think about in terms of the real world. But it DOES have its applications, as do derivatives of higher order than that. So we have the following notation:

$$y''' = f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

$$y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n}$$

$f^2(x)$ means $(f(x))^2$

$\sin^2 x$

$$f^{(2)}(x) = f''(x) = \frac{d^2 f}{dx^2}$$

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